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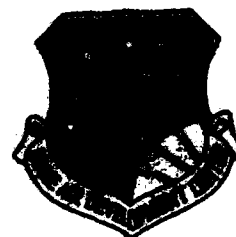
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# **FULL WAVE ANALYSIS FOR SCATTERING CROSS SECTIONS**

University of Nebraska - Lincoln

Ezekiel Bahar

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PART 1: SCATTERING CROSS SECTIONS FOR RANDOM ROUGH SURFACES — FULL WAVE ANALYSIS.

The full wave approach developed earlier to evaluate the radiation fields scattered by deterministic two dimensionally rough surfaces is used here to determine the scattering cross sections for random rough surfaces. The medium below the irregular boundary is characterized by complex permittivity and permeability. For slightly rough surfaces, the full wave solutions for the incoherent scattered fields are shown to be in agreement with the perturbation solution. However, when the major contributions to the scattered fields come from the region of the rough surface around the stationary phase (specular) points, the full wave solutions are in agreement with the physical optics solutions. Thus, the full wave solutions which reduce to the perturbation, the physical optics and the geometrical optics approximations in special cases, precisely determine the limitations of these approximations and reconcile the differences between them.

The full wave solutions satisfy duality, reciprocity and realizability relations in electromagnetic theory and they are invariable under coordinate transformations.

PART 2: SCATTERING CROSS SECTIONS FOR COMPOSITE RANDOM SURFACES — FULL WAVE ANALYSIS.

The full wave approach to rough surface scattering is applied to composite models of rough surfaces. In this work the principal distinguishing features of the individual rough surface is its correlation distance. Thus this model can be applied to scattering by rough seas as well as hilly terrain. It is shown that the full wave approach accounts for both specular scatter and Bragg scattering. The scattering cross section for the composite surface, with two or more roughness scales, is shown to be a weighted sum of the scattering cross sections for the individual rough surface heights. Shadowing effects are accounted for explicitly in the analysis. The full wave solutions satisfy reciprocity, duality and realizability relationships in electromagnetic theory.

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## PART I

SCATTERING CROSS SECTIONS FOR RANDOM ROUGH SURFACES  
--FULL WAVE ANALYSIS

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## Abstract

The full wave approach developed earlier to evaluate the radiation fields scattered by deterministic two dimensionally rough surfaces is used here to determine the scattering cross sections for random rough surfaces. The medium below the irregular boundary is characterized by complex permittivity and permeability. For slightly rough surfaces, the full wave solutions for the incoherent scattered fields are shown to be in agreement with the perturbation solution. However, when the major contributions to the scattered fields come from the region of the rough surface around the stationary phase (specular) points, the full wave solutions are in agreement with the physical optics solutions. Thus, the full wave solutions which reduce to the perturbation, the physical optics and the geometrical optics approximations in special cases, precisely determine the limitations of these approximations and reconcile the differences between them.

The full wave solutions satisfy duality, reciprocity and realizability relations in electromagnetic theory and they are invariable under coordinate transformations.

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## 1. Introduction

The full wave solution for the scattered radiation field by deterministic two dimensionally rough surfaces (Bahar 1980) is applied, in this paper, to problems of scattering and depolarization by random rough surfaces. The results of this analysis are compared with the solutions derived from two general approaches to random rough surface scattering problems: the perturbation technique and the Kirchhoff-Physical Optics approximation (Ishimaru 1978). The perturbation technique, which applies to surfaces that are slightly rough was used by Rayleigh (Strutt 1896) and extended by Rice (1951), Barrick and Peake (1968), Barrick (1970, 1971), Wright (1966) and Rosich and Wait (1977), Valenzuela (1978). The Kirchhoff-Physical Optics approximation technique was applied to surfaces with radii of curvature that are much larger than the wavelength of the electromagnetic excitation (Beckmann and Spizzichino, 1963, Beckmann 1968, Ament 1953).

The principal elements of the full wave approach are (Bahar 1980): (a) Complete expansion of the fields into vertically and horizontally polarized waves. The complete spectrum of the waves consists of the radiation fields (considered here in detail) and the surface and the lateral wave terms (Bahar 1973a,b). (b) Imposition of exact boundary conditions at the irregular interface between two media  $y > h(x,z)$  and  $y < h(x,z)$  characterized by complex electromagnetic parameters  $\epsilon$  and  $\mu$  for  $\exp(i\omega t)$  time harmonic excitations. Thus approximate impedance boundary conditions are not used in this work. (c) Use of Green's theorems to avoid term by term differentiation of the complete expansions. (d) Conversion of Maxwell's equations into rigorous sets of coupled first-order differential equations (generalized telegraphist's equations) for the wave amplitudes. (e) Use of a variable coordinate system that conforms with the local features of the rough surface. Thus there are no restrictions on the height or slope of the rough surface, and both upward and downward scattering of the incident fields are accounted for in the analysis. The effects of shadowing can also be included in the full wave analysis. The full wave solutions are shown to

satisfy duality, reciprocity and realizability relationships in electromagnetic theory (Bahar 1980).

For the convenience of the reader, the full wave solutions for the radiation fields scattered by deterministic two-dimensionally rough surfaces, are summarized in Section 2 since they constitute the starting point of the present analysis. Of particular interest in this work is the bistatic scattering cross section per unit area for rough surfaces (See Appendix A) (Barrick 1970, Ishimaru 1978). The full wave solutions for the scattering cross sections are developed first for slightly rough surfaces in Section 3. In this special case the full wave solutions are shown to be in complete agreement with the scattering cross sections for the incoherent (diffuse) fields (Barrick and Peake 1968).

In Section 4 of this paper, the full wave approach is applied to rough surfaces with normal height distributions. In this section it is shown that if a high frequency stationary phase approximation is made to the full wave solutions, the expression for the scattering cross section reduces to the Kirchhoff--Physical Optics solutions, (Beckmann and Spizzichino 1963, Ishimaru 1978), with the exception that here, consistent with reciprocity, the Fresnel reflection coefficients are evaluated at the stationary phase points, rather than at the angle of incidence with respect to the reference plane.

Except for scattering in the specular direction with respect to the reference plane ( $\theta_0^f = \theta_0^i$ ), the perturbation and the physical optics solutions are not in agreement even for slightly rough surfaces. Since both the perturbation and physical optics solutions are derived here (as special cases) from the full wave solution, the limitations of each of the two special approaches are examined and the differences between these two solutions are reconciled. Thus it is shown that the physical optics solutions cannot be used when the major contributions to the scattered fields do not come from regions of the rough surface around the stationary phase points. As a result, the perturbation solution



(and not the Physical Optics solution) should be used for slightly rough surfaces, even when the radii of curvature for the rough surfaces are much larger than the wavelength.

For very rough random surfaces, it is shown that in agreement with the Physical Optics solution, the scattering cross section is proportional to the probability density function for the rough surface slopes. However, even at high frequencies these solutions cannot be used when the incident or scatter angles are much larger than the mean value of the rough surface slope (see Section 4).

The full wave approach is not limited to slightly rough surfaces or to the special cases when the stationary phase approximations are valid. However, in these two special cases the full wave solutions simplify significantly since they do not depend explicitly on the slopes of the rough surface. In Section 5 the relationships between Physical Optics, Geometrical Optics, Perturbation and the Full Wave solution are summarized.

It is interesting to note that in order to obtain the perturbation solution (as a limiting case of the full-wave solution), it is assumed here that the slope of the rough surface is small but no restrictions are made on the height of the rough surface. Thus, the perturbation solution derived here contains both the incoherent (diffuse) scattered fields (Rice 1951, Barrick and Peake 1968, Barrick 1970, 1971, Wright 1966, Rosich and Wait 1977, Valenzuela 1978) as well as the coherent scattered fields (see Section 3 and Appendix A).

## 2. Formulation of the Problem

The starting point for the present analysis of scattering and depolarization by random rough surfaces is the full wave solution for the scattered radiation field by deterministic, two dimensionally rough surfaces (See Fig. 1):

$$y - h(x, z) = f(x, y, z) = 0 \quad . \quad (1)$$

The incident and scattered radiation fields are decomposed into a complete spectrum of vertically and horizontally polarized components with respect to the reference plane normal to  $\bar{a}_y$ . Denoting the incident and scattered fields by the superscripts  $i$  and  $f$  respectively and the vertically and horizontally polarized components of the fields by the superscripts  $V$  and  $H$  respectively, the electric and magnetic fields  $\bar{E}$  and  $\bar{H}$  can be expressed in matrix notation as follows:

$$G^i = \begin{bmatrix} E^{Vi} \\ E^{Hi} \end{bmatrix} = \eta_0 \begin{bmatrix} H^{Vi} \\ H^{Hi} \end{bmatrix}, \quad G^f = \begin{bmatrix} E^{Vf} \\ E^{Hf} \end{bmatrix} = \eta_0 \begin{bmatrix} H^{Vf} \\ H^{Hf} \end{bmatrix}, \quad (2)$$

where  $\eta_0 = (\mu_0/\epsilon_0)^{1/2}$  is the intrinsic impedance for free space. The full wave solution for  $G^f$  is (Bahar 1980)

$$G^f = G_0 \int_A C_0^{in,f} T^f T^i \exp[ik_0 (\bar{n}^f - \bar{n}^i) \cdot \bar{r}_s] U(\bar{r}_s) d\bar{A} \cdot \bar{n} G^i \\ \equiv G_0 C(\bar{n}^f, \bar{n}^i) G^i \quad . \quad (3)$$

An  $\exp(i\omega t)$  time dependence is assumed and the constant  $G_0$  is given by

$$G_0 = -ik_0 \exp[-ik_0 r^f] / 2\pi r^f, \quad (4)$$

where  $k_0 = \omega(\mu_0/\epsilon_0)^{1/2}$  is the free space wave number and

$$\bar{r}^f = r^f \bar{n}^f = r^f (\sin\theta_0^f \cos\phi^f \bar{a}_x + \cos\theta_0^f \bar{a}_y + \sin\theta_0^f \sin\phi^f \bar{a}_z) \quad (5)$$

is the position vector to the observation point. The position vector

to the source is

$$\vec{r}^i = -r^i \vec{n}^i = -r^i [\sin\theta_o^i \cos\phi^i \vec{a}_x - \cos\theta_o^i \vec{a}_y + \sin\theta_o^i \sin\phi^i \vec{a}_z] , \quad (6)$$

in which  $\vec{n}^i$  is the direction of propagation of the incident waves.

The position vector to the rough surface is

$$\vec{r}_s = x\vec{a}_x + h(x,z)\vec{a}_y + z\vec{a}_z = \vec{r} - f(x,y,z)\vec{a}_y , \quad (7)$$

in which  $f(x,y,z)$  is given by (1). The elementary area of the rough surface is

$$d\vec{A} = \vec{n} dx dy / (\vec{n} \cdot \vec{a}_y) , \quad (8)$$

in which  $\vec{n}$  is the unit vector normal to the rough surface:

$$\begin{aligned} \vec{n} &= \nabla f / |\nabla f| = [-h_x \vec{a}_x + \vec{a}_y - h_z \vec{a}_z] / (h_x^2 + 1 + h_z^2)^{1/2} \\ &= \sin\gamma \cos\delta \vec{a}_x + \cos\gamma \vec{a}_y + \sin\gamma \sin\delta \vec{a}_z , \end{aligned} \quad (9)$$

and

$$h_x = \partial h / \partial x , \quad h_z = \partial h / \partial z . \quad (10)$$

A local Cartesian coordinate system with coordinate surfaces normal to the unit vectors  $\vec{n}_1$ ,  $\vec{n}_2$  and  $\vec{n}_3$  is employed to derive the full wave solution:

$$\vec{n}_1 = \vec{n} \times (\vec{a}_x \times \vec{n}) / |\vec{a}_x \times \vec{n}| , \quad \vec{n}_2 = \vec{n} \text{ and } \vec{n}_3 = (\vec{a}_x \times \vec{n}) / |\vec{a}_x \times \vec{n}| \quad (11)$$

The vertically and horizontally polarized components of the incident and scattered electric and magnetic fields with respect to the local plane, normal to the unit vector  $\vec{n}$ , are denoted by the subscript n.

They are related to the components with respect to the reference plane through the transformations

$$G^{in} = \begin{pmatrix} E_n^{Vi} \\ E_n^{Hi} \end{pmatrix} = T^i G^i = \begin{pmatrix} C_\psi^i & S_\psi^i \\ -S_\psi^i & C_\psi^i \end{pmatrix} \begin{pmatrix} E^{Vi} \\ E^{Hi} \end{pmatrix} , \quad (12)$$

and

$$G^f = \begin{pmatrix} E^{Vf} \\ E^{VH} \end{pmatrix} = T^f G^{fn} = \begin{pmatrix} C_\psi^f & -S_\psi^f \\ S_\psi^f & C_\psi^f \end{pmatrix} \begin{pmatrix} E_n^{Vf} \\ E_n^{Hf} \end{pmatrix}, \quad (13)$$

in which  $C_\psi^i$  and  $S_\psi^i$  are the cosine and sine of the angle between the local plane of incidence and reference plane of incidence normal to the unit vectors  $\bar{a}_{Hi}^n$  and  $\bar{a}_{Hi}^i$ , respectively. Thus they can be expressed in terms of the scalar product and the scalar triple product:

$$C_\psi^i = \cos\psi^i = \bar{a}_{Hi}^n \cdot \bar{a}_{Hi}^i, \quad S_\psi^i = \sin\psi^i = [\bar{a}_{Hi}^n \bar{a}_{Hi}^i \bar{n}^i], \quad (14)$$

where

$$\bar{a}_{Hi}^i = (\bar{n}^i \times \bar{a}_y) / |\bar{n}^i \times \bar{a}_y|, \quad \bar{a}_{Hi}^n = (\bar{n}^i \times \bar{n}) / |\bar{n}^i \times \bar{n}|. \quad (15)$$

Similarly  $C_\psi^f$  and  $S_\psi^f$  are the cosine and sine of the angle between the local plane of scatter and the reference plane of scatter normal to unit vectors  $\bar{a}_{Hf}^n$  and  $\bar{a}_{Hf}^f$ , respectively. Thus

$$C_\psi^f = \cos\psi^f = \bar{a}_{Hf}^n \cdot \bar{a}_{Hf}^f, \quad S_\psi^f = \sin\psi^f = [\bar{a}_{Hf}^n \bar{a}_{Hf}^f \bar{n}^f], \quad (16)$$

where

$$\bar{a}_{Hf}^f = (\bar{n}^f \times \bar{a}_y) / |\bar{n}^f \times \bar{a}_y|, \quad \bar{a}_{Hf}^n = (\bar{n}^f \times \bar{n}) / |\bar{n}^f \times \bar{n}|. \quad (17)$$

The elements  $F^{PQ}$  ( $P, Q=V, H$ ) of the  $2 \times 2$  local scattering matrix  $F(\bar{n}^{fn}, \bar{n}^{in})$  in (3) are (Bahar 1980):

$$C_o^{inFVV} = \frac{2C_o^{infn} [(\mu_r C_1^{infn} \cos(\phi^{fn} - \phi^{in}) - S_o^{infn}) (1 - 1/\epsilon_r) + (1 - \mu_r) \cos(\phi^{fn} - \phi^{in})]}{(C_o^{in} + \eta_r C_1^{in}) (C_o^{fn} + \eta_r C_1^{fn}) (C_o^{in} + C_o^{fn})}, \quad (18a)$$

$$C_o^{inFHH} = \frac{2C_o^{infn} [(\epsilon_r C_1^{infn} \cos(\phi^{fn} - \phi^{in}) - S_o^{infn}) (1 - 1/\mu_r) + (1 - \epsilon_r) \cos(\phi^{fn} - \phi^{in})]}{(C_o^{in} + C_1^{in}/\eta_r) (C_o^{fn} + C_1^{fn}/\eta_r) (C_o^{in} + C_o^{fn})}, \quad (18b)$$

$$C_o^{inFHV} = \frac{-\sin(\phi^{fn} - \phi^{in}) 2C_o^{infn} [(1 - 1/\epsilon_r) C_1^{in} - (1 - 1/\mu_r) C_1^{fn}]}{(C_o^{in} + \eta_r C_1^{in}) (C_o^{fn} + C_1^{fn}/\eta_r) (C_o^{in} + C_o^{fn})}, \quad (18c)$$

$$C_o^{inFVH} = \frac{\sin(\phi^{fn} - \phi^{in}) 2C_o^{infn} [(1 - 1/\mu_r) C_1^{in} - (1 - 1/\epsilon_r) C_1^{fn}]}{(C_o^{in} + C_1^{in}/\eta_r) (C_o^{fn} + \eta_r C_1^{fn}) (C_o^{in} + C_o^{fn})}, \quad (18d)$$

in which the dimensionless quantities  $n_r$ ,  $\eta_r$ ,  $\epsilon_r$  and  $\mu_r$  are the refractive index, relative intrinsic impedance, relative permittivity and relative permeability, respectively,

$$n_r = (\epsilon_1 \mu_1 / \epsilon_0 \mu_0)^{1/2}, \quad \eta_r = \eta_1 / \eta_0 = (\mu_1 / \epsilon_1) / (\mu_0 / \epsilon_0)^{1/2},$$

$$\epsilon_r = \epsilon_1 / \epsilon_0, \quad \mu_r = \mu_1 / \mu_0. \quad (19)$$

The media for  $y > h(x, z)$  and  $y < h(x, z)$  are denoted by the subscripts 0 and 1 respectively. The permittivity and permeability for medium 1 can in general, be complex to account for dissipation. The cosines and sines of the angles of incidence and scatter (with respect to the local coordinate system)  $\theta_0^{in}$  and  $\theta_0^{fn}$  in free space,  $y > h(x, z)$ , are given by:

$$C_0^{in} = \cos \theta_0^{in} = -\vec{n}^i \cdot \vec{n}^f, \quad C_0^{fn} = \cos \theta_0^{fn} = \vec{n}^f \cdot \vec{n}, \quad (20a)$$

$$S_0^{in} = |\vec{n}^i \times \vec{n}|, \quad S_0^{fn} = |\vec{n}^f \times \vec{n}|. \quad (20b)$$

The sines of the corresponding angles in medium 1,  $y < h(x, z)$ , are given by Snell's law:

$$S_1^{in} = \sin \theta_1^{in} = S_0^{in} / n_r, \quad S_1^{fn} = \sin \theta_1^{fn} = S_0^{fn} / n_r. \quad (21)$$

Thus

$$C_1^{in} = \cos \theta_1^{in} = [1 - (S_1^{in})^2]^{1/2}, \quad C_1^{fn} = \cos \theta_1^{fn} = [1 - (S_1^{fn})^2]^{1/2}. \quad (22)$$

The cosine and sine of the angle between the planes of incidence and scatter in the local coordinate system, (1) are given by:

$$\cos(\phi^{fn} - \phi^{in}) = \vec{a}_{Hf}^n \cdot \vec{a}_{Hi}^n, \quad (23a)$$

and

$$\sin(\phi^{fn} - \phi^{in}) = [\vec{a}_{Hf}^n \times \vec{a}_{Hi}^n \cdot \vec{n}]. \quad (23b)$$

The shadow function  $U(\vec{r}_s)$  in (3) is

$$U(\vec{r}_s) = \begin{cases} 1, & \text{illuminated and visible region} \\ 0, & \text{nonilluminated or nonvisible region} \end{cases}. \quad (24)$$

The nonilluminated (shadow) region of the rough surface extends from the locus of points  $\vec{r}_s = \vec{r}_{s1}^i$  to the locus of points  $\vec{r}_s = \vec{r}_{s2}^i$  that satisfy

$$\vec{n}^i \cdot \vec{n}(\vec{r}_{s1}^i) = 0 \quad \text{and} \quad (\vec{r}_{s2}^i - \vec{r}_{s1}^i) \cdot \vec{n}(\vec{r}_{s1}^i) = 0 \quad (25)$$

Similarly, the nonvisible region of the rough surface extends from the locus of points  $\vec{r}_s = \vec{r}_{s1}^f$  to the locus of points  $\vec{r}_s = \vec{r}_{s2}^f$  that satisfy

$$\vec{n}^f \cdot \vec{n}(\vec{r}_{s1}^f) = 0 \quad \text{and} \quad (\vec{r}_{s2}^f - \vec{r}_{s1}^f) \cdot \vec{n}(\vec{r}_{s1}^f) = 0 \quad (26)$$

The full wave solution, (3) satisfies reciprocity, duality and realizability relations in electromagnetic theory (Bahar 1980). Of special interest in this work is the normalized scattering cross section per unit area for rough surfaces (Ishimaru 1978):

$$\sigma^{PQ} = 4\pi(\vec{r}^f)^2 |E^{Pf}|^2 / A_y |E^{Qi}|^2, \quad (27)$$

in which  $A_y$  is the projection of the area of the rough surface  $A$  on the reference plane normal to  $\vec{a}_y$ . Thus for  $P, Q = V, H$

$$\sigma^{PQ} = \frac{k_o^2}{\pi A_y} |C^{PQ}|^2 = \frac{k_o^2}{\pi A_y} \left\{ D^{PQ}(\vec{r}_s) D^{PQ*}(\vec{r}_s') \exp[ik_o(\vec{n}^f - \vec{n}^i) \cdot (\vec{r}_s - \vec{r}_s')] U(\vec{r}_s) U(\vec{r}_s') \frac{dx dy dx' dy'}{(\vec{n} \cdot \vec{a}_y)(\vec{n}' \cdot \vec{a}_y)} \right\}, \quad (28)$$

in which the symbol  $*$  denotes the complex conjugate, and  $D^{PQ}$  are the elements of the matrix  $D$ :

$$D = C_o^{in} T^f F T^i = C_o^{in} \begin{bmatrix} C_\psi^f(F^{VV} C_\psi^i - F^{VH} S_\psi^i) - S_\psi^f(F^{HV} C_\psi^i - F^{HH} S_\psi^i) & C_\psi^f(F^{VV} S_\psi^i + F^{VH} C_\psi^i) - S_\psi^f(F^{HV} S_\psi^i + F^{HH} C_\psi^i) \\ S_\psi^f(F^{VV} C_\psi^i - F^{VH} S_\psi^i) + C_\psi^f(F^{HV} C_\psi^i - F^{HH} S_\psi^i) & S_\psi^f(F^{VV} S_\psi^i + F^{VH} C_\psi^i) + C_\psi^f(F^{HV} S_\psi^i + F^{HH} C_\psi^i) \end{bmatrix} \quad (29)$$

Since  $D^{PQ}(\vec{r}_s)$  is a function of the angles of incidence and scatter in the local coordinate system, (11), it depends explicitly only on the gradient of the rough surface  $Vf$  (9). The exponent

$$\exp[ik_o(\vec{n}^f - \vec{n}^i) \cdot (\vec{r}_s - \vec{r}_s')] = \exp[iv_x(x - x') + iv_z(z - z') + iv_y(h(x, z) - h(x', z'))], \quad (30)$$

depends explicitly only on the height of the rough surface  $h(x,y)$ . The vector  $\bar{v}$  defined in (6.30) is given by

$$\bar{v} = v_x \bar{a}_y + v_y \bar{a}_x + v_z \bar{a}_z = k_o (\bar{n}^f - \bar{n}^i) \quad (31)$$

The shadow function  $U(\bar{r}_s)$  (24) in general, depends on the gradient of the rough surface, through the unit vector  $\bar{n}$  (9), as well as on the height of the rough surface  $h(x,y)$ , (25), (27). However  $U(\bar{r}_s) \neq 1$  only if  $\bar{n}^i \cdot \bar{n} > 0$  or  $\bar{n}^f \cdot \bar{n} < 0$  on portions of the rough surface independent of  $h(x,z)$ . Thus, the shadow function  $U(\bar{r}_s)$  is more sensitive to the gradient of the rough surface than to its height (Sancer 1969, Brown 1978).

The remainder of this paper deals with random rough surfaces for which only the statistics of the height  $h(x,z)$  and its gradient are assumed to be known. Section 3 deals with slightly rough surfaces and the full wave solutions are compared with the perturbational solution (Rice 1951, Barrick and Peake 1968, Barrick 1970, Rosich and Wait 1977). In section 4,  $h(x,z)$  is assumed to be normally distributed and no restrictions are made on the variance of the rough surface. These solutions are compared with both the perturbational and physical optics solutions (Beckmann and Spizzichino 1963).

### 3. Incoherent Scattering Cross Section Per Unit

#### Area For Slightly Rough Surfaces.

For the incoherent, diffuse field, the scattering cross section for unit cross sectional area is given by (Ishimaru 1978):

$$\langle \sigma^{PQ} \rangle = 4\pi (r^f)^2 \langle |E^{Pf} - \langle E^{Pf} \rangle|^2 \rangle / A_y |E^{Qi}|^2, \quad (32)$$

in which the symbol  $\langle \rangle$  denotes the statistical average. For slightly rough surfaces  $\tan \gamma \ll 1$  and  $\bar{n}$  is set equal to  $\bar{a}_y$  in (3). Thus

$$T^i \rightarrow I, \quad T^f \rightarrow I, \quad U(\bar{r}_s) \rightarrow 1, \quad \bar{n} \cdot \bar{a}_y \rightarrow 1, \quad (33)$$

in which  $I$  is the  $2 \times 2$  identity matrix. Thus, the elements of the matrix  $D$  (29) are no longer functions of position and may be extracted from the integral, (32).

The incoherent scattering cross section, (32) reduces to

$$\langle \sigma_o^{PQ} \rangle = \frac{k_o^2 |D_o^{PQ}|^2}{\pi} \mathcal{J} , \quad (34)$$

in which

$$D_o^{PQ} = (C_o^{in, PQ})_{n=a_y} , \quad (35)$$

and

$$\mathcal{J} = \frac{1}{A_y} \int_A \exp[i v_x (x-x') + i v_z (z-z')] [\chi_2(v_y, -v_y) - |\chi(v_y)|^2] dx dy dx' dy' . \quad (36)$$

The characteristic function  $\chi(v_y)$  and the joint characteristic function  $\chi(v_y, -v_y)$  are defined in terms of the probability density function  $W(h)$  and joint probability density function  $W(h, h')$ :

$$\chi(v_y) = \int_{-\infty}^{\infty} \exp(i v_y h) W(h) dh , \quad (37)$$

and

$$\chi(v_y, -v_y) = \int_{-\infty}^{\infty} \exp[i v_y (h-h')] W(h, h') dh dh' . \quad (38)$$

Thus for slightly rough surfaces

$$\exp(i v_y h) \approx 1 + i v_y h - \frac{v_y^2}{2} h^2 , \quad (39)$$

and

$$\chi(v_y) \approx 1 - \frac{1}{2} v_y^2 \langle h^2 \rangle = 1 - \frac{1}{2} v_y^2 \sigma_o^2 , \quad (40)$$

in which  $\sigma_o$  is the variance of  $h$  and  $\langle h \rangle = 0$ . Similarly,

$$\chi_2(v_y, -v_y) \approx 1 - \frac{v_y^2}{2} \langle h^2 \rangle + \frac{v_y^2}{2} \langle hh' \rangle = 1 - \frac{v_y^2}{2} \sigma_o^2 (1-C) , \quad (41)$$

in which  $C$  is the normalized correlation coefficient. Assuming that the rough surface is statistically homogeneous and isotropic, the surface height correlation function  $\langle hh' \rangle$  depends only on

$$x_d = x-x' \quad \text{and} \quad z_d = z-z' . \quad (42)$$

Thus if the correlation distance is much smaller than the width of the illuminated surface, (36) reduces to



$$\mathcal{J} = \int_{-\infty}^{\infty} \exp[i v_x x_d + i v_z z_d] \langle hh' \rangle v_y^2 dx_d dz_d = \pi^2 v_y^2 W(v_x, v_z) , \quad (43)$$

in which  $W(v_x, v_z)$  is the spectral density of the rough surface height function (Barrick 1970, Ishimaru 1978). Thus the incoherent scattering cross section for slightly rough surfaces is given by

$$\langle \sigma_o^{PQ} \rangle = \pi k_o^4 W(v_x, v_z) \left| C_o^{in, PQ} (\cos \theta_o^f + \cos \theta_o^i) \right|_{\vec{n} \rightarrow \vec{a}_y}^2 . \quad (44)$$

The expression (44) is precisely equal to the perturbation solution for the scattering cross section for slightly rough surfaces (Barrick and Peake 1968, Barrick 1970). In the notation used for the perturbation solution

$$[C_o^{in, PQ} (\cos \theta_o^f + \cos \theta_o^i)]_{\vec{n} \rightarrow \vec{a}_y} = \pm 2 \cos \theta_o^f \cos \theta_o^i \alpha^{PQ} , \quad (45)$$

in which the upper and lower signs are used for  $Q = H$  and  $V$ , respectively.

For the perfectly conducting case the scattering cross section is given in matrix form by

$$\langle \sigma_o \rangle = 4\pi k_o^4 W(v_x, v_z) \begin{bmatrix} [\sin \theta_o^i \sin \theta_o^f - \cos(\phi^f - \phi^i)]^2 & [\cos \theta_o^i \sin(\phi^f - \phi^i)]^2 \\ [\cos \theta_o^f \sin(\phi^f - \phi^i)]^2 & [\cos \theta_o^i \cos \theta_o^f \cos(\phi^f - \phi^i)]^2 \end{bmatrix} . \quad (46)$$

For backscatter  $\theta_o^f = \theta_o^i$ ,  $\phi^f - \phi^i = \pi$  (46) reduces to

$$\langle \sigma_o \rangle_B = 4\pi k_o^4 W(2k_o \sin \theta_o^i, 0) \begin{bmatrix} (\sin^2 \theta_o^i + 1)^2 & 0 \\ 0 & \cos^4 \theta_o^i \end{bmatrix} . \quad (47)$$

#### 4. Incoherent Scattering Cross Section For Rough Surfaces With Normal Distributions

When the slope of the rough surface is not small, the incoherent scattering cross section for high frequencies, (32) can be written as follows (see Appendix A):

$$\langle \sigma^{PQ} \rangle = \frac{k_0^2}{\pi} \int_{A_y} \langle S^{PQ} \rangle \chi_2(v_y, -v_y) |\chi(v_y)|^2 \exp[iv_x x_d + iv_z z_d] dx_d dz_d, \quad (48a)$$

in which

$$\langle S^{PQ} \rangle = \langle \frac{D^{PQ}(\vec{r}_s) D^{PQ*}(\vec{r}_s') U(\vec{r}_s) U(\vec{r}_s')}{(\vec{n} \cdot \vec{a}_y) (\vec{n}' \cdot \vec{a}_y)} \rangle = \langle \frac{|D^{PQ}(\vec{r}_s) U(\vec{r}_s)|^2}{(\vec{n} \cdot \vec{a}_y)} \rangle, \quad (48b)$$

To derive (48) it is assumed that the surface height and gradient are uncorrelated (which is true for Gaussian surfaces) and that the probability density function for the shadow function  $U(\vec{r}_s)$  depends only upon the gradient of the rough surface (see Section 2).

As in Section 3, the rough surface is assumed to be statistically homogeneous and isotropic; thus  $\langle S^{PQ} \rangle$  and  $\chi_2$  depend only on  $x_d$  and  $z_d$ , (42).

For normally distributed surfaces the probability density function and the joint probability density functions  $W(h)$  and  $W(h, h')$  are given by:

$$W_0(h) = \frac{1}{(2\pi)^{1/2} \sigma_0} \exp\left(-\frac{h^2}{2\sigma_0^2}\right) \quad (49)$$

and

$$W(h, h') = \frac{1}{2\pi \sigma_0^2 (1-C^2)^{1/2}} \exp\left[-\frac{h^2 - 2Chh' + h'^2}{2\sigma_0^2 (1-C^2)}\right], \quad (50)$$

in which  $\sigma_0$  is the variance and  $C$  is the correlation coefficient. The corresponding characteristic functions are

$$\chi(v_y) = \exp(-v_y^2 \sigma_0^2 / 2), \quad (51)$$

and

$$\chi_2(v_y, -v_y) = \exp[-v_y^2 \sigma_0^2 (1-C)] \quad (52)$$

Thus it follows that

$$\chi_2(v_y, -v_y) = |\chi(v_y)|^2 = \exp(-v_y^2 \sigma_0^2) [\exp(v_y^2 \sigma_0^2 C) - 1], \quad (53)$$

and the surface height correlation function is

$$\langle h(x, y) h'(x', y') \rangle = \sigma_0^2 C \quad (54)$$

Thus for slightly rough surfaces  $[(v_y \sigma_o)^2 \ll 1, \bar{n} \approx \bar{a}_y]$ , (54) reduces to

$$\chi_2(v_y, -v_y) = |\chi(v_y)|^2 \approx v_y^2 \langle hh' \rangle, \quad (55a)$$

$$\langle S^{PQ} \rangle = [S^{PQ}]_{\bar{n} \rightarrow \bar{a}_y} = |D_o^{PQ}|^2, \quad (55b)$$

and  $\langle S^{PQ} \rangle$  is given by the perturbation solution, (44).

If it is not assumed a priori that the surface is slightly rough,  $\langle S^{PQ} \rangle$  can be expressed approximately as follows for high frequencies:

$$\langle S^{PQ} \rangle = \frac{k_o^2}{\pi} S_\infty^{PQ} \mathcal{J}, \quad (56a)$$

in which

$$\mathcal{J} = 2\pi \int_0^\infty J_0(v_{xz} \rho) \exp(-v_y^2 \sigma_o^2) [\exp(v_y^2 \sigma_o^2 C) - 1] \rho d\rho, \quad (56b)$$

where  $J_0$  is the Bessel Function of the first kind, and

$$v_{xz} = (v_x^2 + v_z^2)^{1/2} \quad \text{and} \quad \rho = (x_d^2 + z_d^2)^{1/2}. \quad (57)$$

The correlation coefficient  $C$  is only a function of  $\rho$ .

Assuming a correlation coefficient of the form

$$C(\rho) = \exp(-\rho^2/\ell^2), \quad (58)$$

where  $\ell$  is the correlation distance, it can be shown that (Beckmann and Spizzichino 1963):

$$\mathcal{J} = \begin{cases} \pi (v_y \sigma_o \ell)^2 \exp(-v_y^2 \sigma_o^2 - v_{xz}^2 \ell^2/4) = \pi^2 v_y^2 W(v_x, v_z), & (v_y \sigma_o)^2 \ll 1, \\ \pi \ell^2 \exp(-v_y^2 \sigma_o^2) \sum_{m=1}^{\infty} \frac{(v_y \sigma_o)^{2m}}{m! m} \exp(-v_{xz}^2 \ell^2/4), & v_y \sigma_o \approx 1, \\ \frac{\pi \ell^2}{v_y^2 \sigma_o^2} \exp[-(v_{xz} \ell/2v_y \sigma_o)^2], & (v_y \sigma_o)^2 \gg 1. \end{cases} \quad (59)$$

In (56)  $S^{PQ}$  is replaced by its value  $S_\infty^{PQ}$  at the stationary phase points of the integrand in (3). Thus (Bahar 1980):

$$S_\infty^{PQ} = [\cos \theta_o^i F_3(\theta_o^i, \theta_o^f, \phi^f - \phi^i)]^2 |R_{1o}^P(\theta_o^s)|^2 \delta_{PQ}. \quad (60)$$

In view of the Kronecker delta,  $\delta_{PQ}$ ,  $S_{\omega}^{PQ}$  vanishes for  $P \neq Q$ .

In (60)  $\theta_o^s$  is the local angle of incidence and scatter at the specular points. Thus,

$$\begin{aligned} \cos^2 \theta_o^s &= (1 - \bar{n}^f \cdot \bar{n}^i) / 2 \\ &= [1 + \cos \theta_o^i \cos \theta_o^f - \sin \theta_o^i \sin \theta_o^f \cos(\phi^f - \phi^i)] / 2. \end{aligned} \quad (61)$$

The Fresnel reflection coefficients  $R_{10}^P(\theta_o^s)$  ( $P=V,H$ ) are evaluated at the stationary phase points where  $\theta_o^i \rightarrow \theta_o^f \rightarrow \theta_o^s$  and not at the angle of incidence  $\theta_o^i$  as done in Beckmann and Spizzichino, 1963. (Ishimaru, 1978). The coefficient  $F_3$  is precisely the expression derived for the Physical Optics solution (Beckmann and Spizzichino 1963)

$$F_3(\theta_o^i, \theta_o^f, \phi^f - \phi^i) = \frac{2 \cos^2 \theta_o^s}{\cos \theta_o^i (\cos \theta_o^f + \cos \theta_o^i)} \quad (62)$$

Thus the high frequency approximation for  $\langle \sigma_{\omega}^{PQ} \rangle$  is given by

$$\langle \sigma_{\omega}^{PQ} \rangle = \frac{k_o^2}{\pi} \left[ \frac{2 \cos^2 \theta_o^s}{\cos \theta_o^f + \cos \theta_o^i} \right]^2 |R_{10}^P(\theta_o^s)|^2 \delta_{PQ} \quad (63)$$

For the perfectly conducting case  $|R_{10}^P| = 1$  and  $\langle \sigma_{\omega}^{PP} \rangle$  is independent of polarization. Thus the full wave solution for the incoherent scattering cross section reduces to previously derived Physical Optics results when the following conditions are satisfied.

(a) For high frequencies, the principal contributions to the incoherent scattered field come from the stationary phase points of the integrand in (3), (Bahar 1980). In this case  $\bar{n}$  can be replaced by its value  $\bar{n}_s$  at the specularly oriented portions of the rough surface:

$$\bar{n}_s = (\bar{n}^f - \bar{n}^i) / |\bar{n}^f - \bar{n}^i| \quad (64)$$

(b) The vectors  $\bar{n}^f$ ,  $\bar{n}^i$  and  $\bar{a}_y$  are coplanar. Thus

$$[\bar{n}^f \bar{a}_y \bar{n}_s] = [\bar{n}^i \bar{a}_y \bar{n}_s] = [\bar{n}^i \bar{a}_y \bar{n}^f] = 0 \quad (65)$$

and at the stationary phase points

$$\psi^f = \psi^i = 0 \quad (66)$$

In this case the local and reference planes of incidence and scatter are identical:

$$T^i = T^f = I \quad , \quad (67)$$

where  $I$  is the identity matrix

(c) The phase of the integrand in (3) varies rapidly for values of  $\bar{r}_s$  in the shadow region, thus the value of the integral does not change significantly on replacing  $U(\bar{r}_s)$  by unity everywhere on the surface.

For slightly rough surfaces (63) reduces to

$$\langle \sigma_{\infty}^{PP} \rangle = 4\pi k_o^4 |R_{10}^P(\theta_o^s)|^2 \cos^4 \theta_o^s W(v_x, v_z) \quad . \quad (68)$$

The above expression which satisfies reciprocity, is in agreement with the Physical Optics solution given by Ishimaru (1978) with the exception that here the reflection coefficient is evaluated at the specular angle (as required by the stationary phase condition) and not at the angle of incidence. For backscatter ( $\bar{n}^f = -\bar{n}^i = \bar{n}_s$ ,  $\cos \theta_o^s = 1$ ) (68) reduces to

$$\langle \sigma_{\infty}^{PP} \rangle_B = 4\pi k_o^4 |R_{10}^P(0)|^2 W(2k_o \sin \theta_o^i, 0) \quad . \quad (69)$$

Note that for slightly rough surfaces the perturbation solution (44) is not in agreement with the Physical Optics solution, (68) except at normal incidence. This is because even at high frequencies condition (a) is not satisfied for slightly rough surfaces except near normal incidence. For large angles of incidence, there are no stationary phase (specular) points on the slightly rough surface, thus even for high frequencies, the Physical Optics solution cannot, in general, be used for slightly rough surfaces when  $\frac{\pi}{2} - \theta_o^i$  is smaller than the mean value of the slope of the rough surface  $\beta_o$ , where

$$\tan \beta_o = 2\sigma_o / \ell \quad . \quad (70)$$

For very rough surfaces,  $[(v_y \sigma_o)^2 \gg 1]$ , the scattering cross section is given by:

$$\begin{aligned} \langle \sigma_{\omega}^{PP} \rangle &= \frac{4k^2}{\sigma_0^2} \left( \frac{\cos \theta_0^s}{\cos \theta_0^f + \cos \theta_0^i} \right)^4 |R_{10}^P(\theta_0^s)|^2 \exp[-(v_{xz} l / 2v_{y0})^2] \\ &= \pi \sec^4 \gamma_s p(h_{xsp}, h_{zsp}) |R_{10}^P(\theta_0^s)|^2, \end{aligned} \quad (71a)$$

in which  $p(h_{xsp}, h_{zsp})$  is the joint probability density function for the slopes,

(9), at the stationary phase points:

$$p(h_{xsp}, h_{zsp}) = \frac{1}{2\pi \sigma_s^2} \exp \left[ -\frac{h_{xsp}^2 + h_{zsp}^2}{2\sigma_s^2} \right] = \frac{1}{2\pi \sigma_s^2} \exp \left[ -\frac{\tan^2 \gamma_s}{2\sigma_s^2} \right], \quad (71b)$$

and

$$\sec^2 \gamma_s = 1 + \tan^2 \gamma_s = 1/(\bar{a}_y \cdot \bar{n}_s)^2 = [2\cos \theta_0^s / (\cos \theta_0^f + \cos \theta_0^i)]^2, \quad \sigma_s^2 = 2\sigma_0^2/l^2. \quad (71c)$$

Thus for backscatter ( $\theta_0^s = 0$ ,  $\phi^i = 0$ ,  $\phi^f = \pi$ ), (71) reduces to

$$\langle \sigma_{\omega}^{PP} \rangle_B = \frac{|R_{10}^P(0)|^2}{\cos^4 \theta_0^i \tan^2 \beta_0} \exp \left[ -\frac{\tan^2 \theta_0^i}{\tan^2 \beta_0} \right] = \frac{\pi}{\cos^4 \theta_0^i} |R_{10}^P(0)|^2 p(\tan \theta_0^i, 0), \quad (72)$$

in which  $R_{10}^P(0)$  is the Fresnel reflection coefficient for normal incidence.

Thus the full wave solution, (43) is in agreement with the earlier high frequency results (Barriek and Peake 1968). However, for very rough surfaces the Physical Optics solution, (72) cannot be used near grazing angles, even for very high frequencies, since, in these cases assumptions (a) and (c) are not satisfied, and shadowing effects become important.

For the Physical Optics, or the perturbation approximations, derived in sections 3 and 4, it is not necessary to know the complete statistics of the gradient  $\nabla f$  of the rough surface. However, when these approximations are not valid, it is necessary to determine the probability density function of the gradient in order to evaluate the scattering cross sections  $\langle \sigma^{PQ} \rangle$  (48).

5. Relationships Between Physical Optics, Geometrical Optics,  
Perturbation Theory and the Full Wave Approach

For very high frequencies (3) can be written as

$$G^f = G_o \int_A \left[ D(\bar{r}_s) U(\bar{r}_s) \right]_{\bar{n}_s} \exp[i\bar{v} \cdot \bar{r}_s] \frac{dx dz}{\bar{n}_s \cdot \bar{a}_y}, \quad (73)$$

where  $\bar{n}$  is replaced by  $\bar{n}_s$ , its value at the stationary phase points (60),  $\bar{v}$  is given by (31) and  $G_o$  is defined by (4). Equation (73) is the physical optics approximation of (3). To obtain the corresponding geometrical optics approximation, start by expanding  $\bar{v} \cdot \bar{r}_s$  about its value at a stationary phase point (where  $\bar{r}_s = \bar{r}_{so}$ ). Thus

$$\begin{aligned} \bar{v} \cdot \bar{r}_s &= v_x x + v_z z + v_y [h_o + h_{xo}(x-x_o) + h_{zo}(z-z_o)] \\ &+ \frac{v_y}{2} [h_{xxo}(x-x_o)^2 + 2h_{xzo}(x-x_o)(z-z_o) + h_{zz}(z-z_o)^2] \\ &= \bar{v} \cdot \bar{r}_{so} + \frac{v_y}{2} [h_{xxo}(x-x_o)^2 + 2h_{xzo}(x-x_o)(z-z_o) + h_{zz}(z-z_o)^2] \end{aligned} \quad (74)$$

in which

$$\left. \begin{aligned} h_{xo} &= (\partial h / \partial x)_{\bar{r}_{so}}, \quad h_{xxo} = (\partial^2 h / \partial x^2)_{\bar{r}_{so}}, \quad h_{xzo} = (\partial^2 h / \partial x \partial z)_{\bar{r}_{so}}, \\ h_{zo} &= (\partial h / \partial z)_{\bar{r}_{so}} \quad \text{and} \quad h_{zzo} = (\partial^2 h / \partial z^2)_{\bar{r}_{so}}, \end{aligned} \right\} \quad (75)$$

and

$$v_x + v_y h_{xo} = 0, \quad v_z + v_y h_{zo} = 0 \quad (76)$$

Using a principal axis coordinate transformation about the stationary phase point,  $\bar{r}_{so}$ , the geometrical optics contribution from the neighborhood of this point can be expressed as (Barrick 1970)

$$\begin{aligned} G_{go}^f &= G_o D(\bar{r}_{so}) U(\bar{r}_{so}) \exp[i\bar{v} \cdot \bar{r}_{so}] \\ &\cdot \int_{-\infty}^{\infty} \exp \left[ \frac{iv_y p}{2} \left( h_{xxp} x_p^2 + h_{zzp} z_p^2 \right) \right] dx_p dz_p \end{aligned}$$

$$\begin{aligned}
&= G_o D(\vec{r}_{so}) U(\vec{r}_{so}) \exp[i\vec{v} \cdot \vec{r}_{so}] \left( \frac{-2\pi i}{v_{yp}} \right) \sqrt{r_{1p} r_{2p}} \\
&= \left( \frac{-\pi i}{k_o} \right) G_o \exp[i\vec{v} \cdot \vec{r}_{so}] \sqrt{r_{1p} r_{2p}} \begin{bmatrix} R_{1o}^V(\theta_o^S) & 0 \\ 0 & R_{1o}^H(\theta_o^S) \end{bmatrix} U(\vec{r}_{so}) , \quad (77)
\end{aligned}$$

The integrals in (77) are identified with the Fresnel Integrals (Abramowitz and Stegun 1964) and  $r_{1p} = \frac{-1}{h_{xyp}}$ ,  $r_{2p} = \frac{-1}{h_{zyp}}$  are the principal radii of curvature (convex side facing observer) at the stationary phase point, ( $v_{yp} = 2k_o \cos \theta_o^S$ ) and  $R_{1o}^P(\theta_o^S)$  is the Fresnel reflection coefficient for the specular angle  $\theta_o^S$ , (65). Provided that this stationary phase point is visible and illuminated [ $U(\vec{r}_{so}) = 1$ ] its contribution to the scattering cross section (28) is given by:

$$\sigma_{go}^{PP} = \frac{\pi r_{1p} r_{2p}}{A_y} \left| R_{1o}^P(\theta_o^S) \right|^2 \quad (78)$$

To determine the average scattering cross section per unit area assume that the principal radii of curvature ( $r_{1p}, r_{2p}$ ) at the stationary phase (specular) points and the location of these points are independent random variables. Furthermore for very rough surfaces the phase  $\vec{v} \cdot \vec{r}_{so}$  can be assumed to be uniformly distributed from  $-\pi$  to  $\pi$ , (Beckmann and Spizzichino 1963). Thus it follows from (77) that

$$\langle \sigma_{go}^{PP} \rangle = \pi \langle r_{1p} r_{2p} N \rangle \left| R_{1o}^P(\theta_o^S) \right|^2 P_2(\vec{n}^f, \vec{n}^i | \vec{n}_s) , \quad (79)$$

in which  $\langle r_{1p} r_{2p} N \rangle$  is the average of the product of the principal radii of curvature and  $N$  the number of specular points per unit area. The probability that a specular point (with slope  $\vec{n} = \vec{n}_s$ ) will be both illuminated and visible is given by  $P_2(\vec{n}^f, \vec{n}^i | \vec{n}_s)$ . The expression for  $P_2$  has been given by Sancer (1969) for rough surfaces with normal distributions. The physical optics result derived by Kodis (1966) for perfectly conducting surfaces is

$$\langle \sigma_{go}^{PP} \rangle = \pi \langle r_{1p} r_{2p} \rangle \langle N \rangle \quad (80)$$



Barrick (1968) determined the relationships between the surface statistics and  $\langle r_{1p} r_{2p} N \rangle$  and has shown that (80) reduces to

$$\langle \sigma^{PP} \rangle = \pi \sec^4 \gamma_s p(h_{xsp}, h_{zsp}) \quad , \quad (81)$$

which is in agreement with (71). In the derivation of (81), however, it is not necessary to assume that the rough surface is normally distributed. Furthermore, it is incorrect to assume as implied by (80) that  $r_{1p} r_{2p}$  and  $N$  are statistically independent.

It has been shown that in order to obtain the physical optics solution (that has appeared previously) from the full wave solution, the coefficient of  $\exp[i\vec{v} \cdot \vec{r}_s]$  in the integrand of (3) is replaced by its value at the stationary phase points ( $\vec{n} = \vec{n}_s$ ) (Section 4). Furthermore, to obtain the geometrical optics solutions, the full wave integral of (3) is evaluated using the stationary phase (or steepest descent) method. It has also been shown (Section 3) that for slightly rough surfaces, the full wave solution reduces to the perturbation solution upon replacing the unit vector normal to the rough surface,  $\vec{n}$ , by the unit vector normal to the reference surface,  $\vec{a}_y$ , in the coefficient of  $\exp[i\vec{v} \cdot \vec{r}_s]$ , (3).

## 6. Concluding Remarks

The full wave approach has been applied to problems of scattering and depolarization of radio waves by random rough surfaces. The general, full wave, expression for the scattering cross sections per unit area is given in Appendix A for the incoherent and coherent fields. In Section 3, which deals with slightly rough surfaces, the full wave solutions are shown to reduce to the perturbation solution. For high frequencies it is shown that when the stationary phase approximation is valid, the full wave solutions reduce to the physical optics--Kirchoff solution. Thus the two general approaches applied to scattering by random rough surfaces, perturbation and physical optics, are derived here as

special cases of the full wave solution. The full wave approach precisely determines the limitations of the earlier approaches as well as reconciles the differences between them. Thus, physical optics, geometric optics and perturbation theory are all special cases of the full wave approach (Sec. 5).

The full wave solution which is invariant under coordinate transformations also satisfies reciprocity, duality and realizability in electromagnetic theory (Bahar 1980).

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## Appendix A

Substituting the full wave solution (3) into (32) and assuming that the rough surface height and gradient are uncorrelated and that the probability density function for the shadow function  $U(\vec{r}_s)$  depends only upon the gradient of the rough surface, the expression for the incoherent scattering cross section reduces to

$$\langle \sigma^{PQ} \rangle = \frac{k_o^2}{\pi} \int_{-\infty}^{\infty} \langle S^{PQ} \rangle \chi_2(\vec{v}_y, \vec{v}_y) - \left| \left\langle \frac{D^{PQ}U}{\vec{n} \cdot \vec{a}_y} \right\rangle \chi(\vec{v}_y) \right|^2 \exp[i\vec{v}_x x_d + i\vec{v}_z z_d] dx_d dz_d \quad (A.1)$$

For simplicity the rough surface is also assumed to be statistically homogeneous and isotropic, thus the surface height correlation function  $\langle h(x,z)h(x',z') \rangle$  depends only upon the distances

$$\vec{r}_d = (x-x')\vec{a}_x + (z-z')\vec{a}_z = x_d\vec{a}_x + z_d\vec{a}_z \quad (A.2)$$

In (A.1)  $\chi$  and  $\chi_2$  are the characteristic function (37) and the joint characteristic function (38). Furthermore,

$$S^{PQ} = D^{PQ}(\vec{r}) D^{PQ*}(\vec{r}') U(\vec{r}) U(\vec{r}') / (\vec{n} \cdot \vec{a}_y) (\vec{n}' \cdot \vec{a}_y) \quad (A.3)$$

in which  $D^{PQ}$  is given by (29) and  $U$  is the shadow function (24). The symbol  $\langle \rangle$  in (48) denotes the statistical average, thus in order to evaluate  $\langle S^{PQ} \rangle$  for the general case, it is necessary to multiply  $S^{PQ}$  by the joint probability density function  $p(\vec{n}, U)$  and integrate with respect to  $h_x, h_z$  (7) and  $U$ . To facilitate the evaluation of (A.1), it is rewritten as follows,

$$\begin{aligned} \langle \sigma^{PQ} \rangle = \frac{k_o^2}{\pi} \int_{-\infty}^{\infty} \left[ \langle S^{PQ} \rangle \left\{ \chi_2 - |\chi|^2 \right\} + |\chi|^2 \left\{ \langle S^{PQ} \rangle - \left| \left\langle \frac{D^{PQ}U}{\vec{n} \cdot \vec{a}_y} \right\rangle \right|^2 \right\} \right] \\ \cdot \exp[i\vec{v}_x x_d + i\vec{v}_z z_d] dx_d dz_d \end{aligned} \quad (A.4)$$

For high frequencies it can be shown (Sancer 1969, Brown 1978), that

$$\langle S^{PQ} \rangle \rightarrow \left\langle \left| \frac{D^{PQ}U}{\vec{n} \cdot \vec{a}_y} \right|^2 \right\rangle \quad (A.5)$$

Thus (A.4) reduces to

$$\begin{aligned} \langle \sigma^{PQ} \rangle = & \frac{k_o^2}{\pi} \int_{-\infty}^{\infty} \left\langle \left| \frac{D^{PQ_U}}{\bar{n} \cdot \bar{a}_y} \right|^2 \right\rangle \left\{ \chi_2 - |\chi|^2 \right\} \exp[i v_x x_d + i v_z z_d] dx_d dz_d \\ & + \frac{k_o^2}{\pi} \int_{-\infty}^{\infty} |\chi|^2 \left\{ \left\langle \left| \frac{D^{PQ_U}}{\bar{n} \cdot \bar{a}_y} \right|^2 \right\rangle - \left| \left\langle \frac{D^{PQ_U}}{\bar{n} \cdot \bar{a}_y} \right\rangle \right|^2 \right\} \exp[i v_x x_d + i v_z z_d] dx_d dz_d \end{aligned} \quad (A.6)$$

For slightly rough surfaces (perturbation solution)

$$\left[ \frac{D^{PQ}}{\bar{n} \cdot \bar{a}_y} \right] \rightarrow \left[ \frac{D^{PQ}}{\bar{n} \cdot \bar{a}_y} \right]_{\bar{n} \rightarrow \bar{n}_s} = D_o^{PQ} \quad (A.7)$$

where  $D_o^{PQ}$  is given by (35). Furthermore, when the stationary phase approximations are valid (see Section 4)

$$\left[ \frac{D^{PQ}}{\bar{n} \cdot \bar{a}_y} \right] \rightarrow \left[ \frac{D^{PQ}}{\bar{n} \cdot \bar{a}_y} \right]_{\bar{n} \rightarrow \bar{n}_s} \quad (A.8)$$

where  $\left| \frac{D^{PQ}}{\bar{n} \cdot \bar{a}_y} \right|_{\bar{n}_s}$  is given by (64). Thus if  $|\chi|^2 \ll 1$  or when either the physical optics or perturbation approximations are valid, the second integral in (A.6) can be neglected and  $\langle \sigma^{PQ} \rangle$  reduces to (48).

The scattering cross section for the coherent fields  $\langle \sigma^{PQ} \rangle^c$  corresponds to the last term in (48). Thus

$$\begin{aligned} \langle \sigma^{PQ} \rangle^c = & \frac{k_o^2}{\pi A_y} \left| \int_{-\infty}^{\infty} \chi(v_y) \left\langle \frac{D^{PQ}}{\bar{n} \cdot \bar{a}_y} \right\rangle \exp[i v_x x + i v_z z] dx dz \right|^2 \\ = & \frac{k_o^2 A_y}{\pi} \left| \chi(v_y) \left\langle \frac{D^{PQ}}{\bar{n} \cdot \bar{a}_y} \right\rangle \right|^2 \text{sinc}(v_x L_x) \text{sinc}(v_z L_z) \end{aligned} \quad (A.9)$$

in which  $\text{sinc}(u) = \sin(u)/u$  and the projection of the rough surface on the xz plane is given by

$$A_y = 2L_x \cdot 2L_z \quad (A.10)$$

Thus for the specular case  $v_x = v_z = 0$

$$\langle \sigma^{PQ} \rangle_s^c = \frac{k_o^2 A_y}{\pi} \left| \chi(v_y) \left\langle \frac{D^{PQ}}{\bar{n} \cdot \bar{a}_y} \right\rangle \right|^2 \quad (A.11)$$

The total scattering cross section corresponds to the first term in (A.1). Thus

$$\langle \sigma^{PQ} \rangle^T = \frac{k_o^2}{\pi A_y} \int_{-\infty}^{\infty} \langle S^{PQ} \rangle \chi_2(v_y, -v_y) \exp[i v_x (x-x') + i v_z (z-z')] dx dz dx' dz' \quad (A.12)$$

However, except for very rough surfaces, where  $\langle \sigma^{PQ} \rangle^c$ , (A.11) can be neglected, the total scattering cross section  $\langle \sigma^{PQ} \rangle^T$  is evaluated by summing the incoherent and coherent terms (A.6) and (A.11) respectively rather than by directly evaluating (A.12) (Beckmann and Spizzichino 1963). Thus,

$$\langle \sigma^{PQ} \rangle^T = \langle \sigma^{PQ} \rangle^c + \langle \sigma^{PQ} \rangle \quad (A.13)$$

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A-4

Figure Caption

Fig. 1. Plane of incidence, scattering plane and reference  $x, z$  plane.

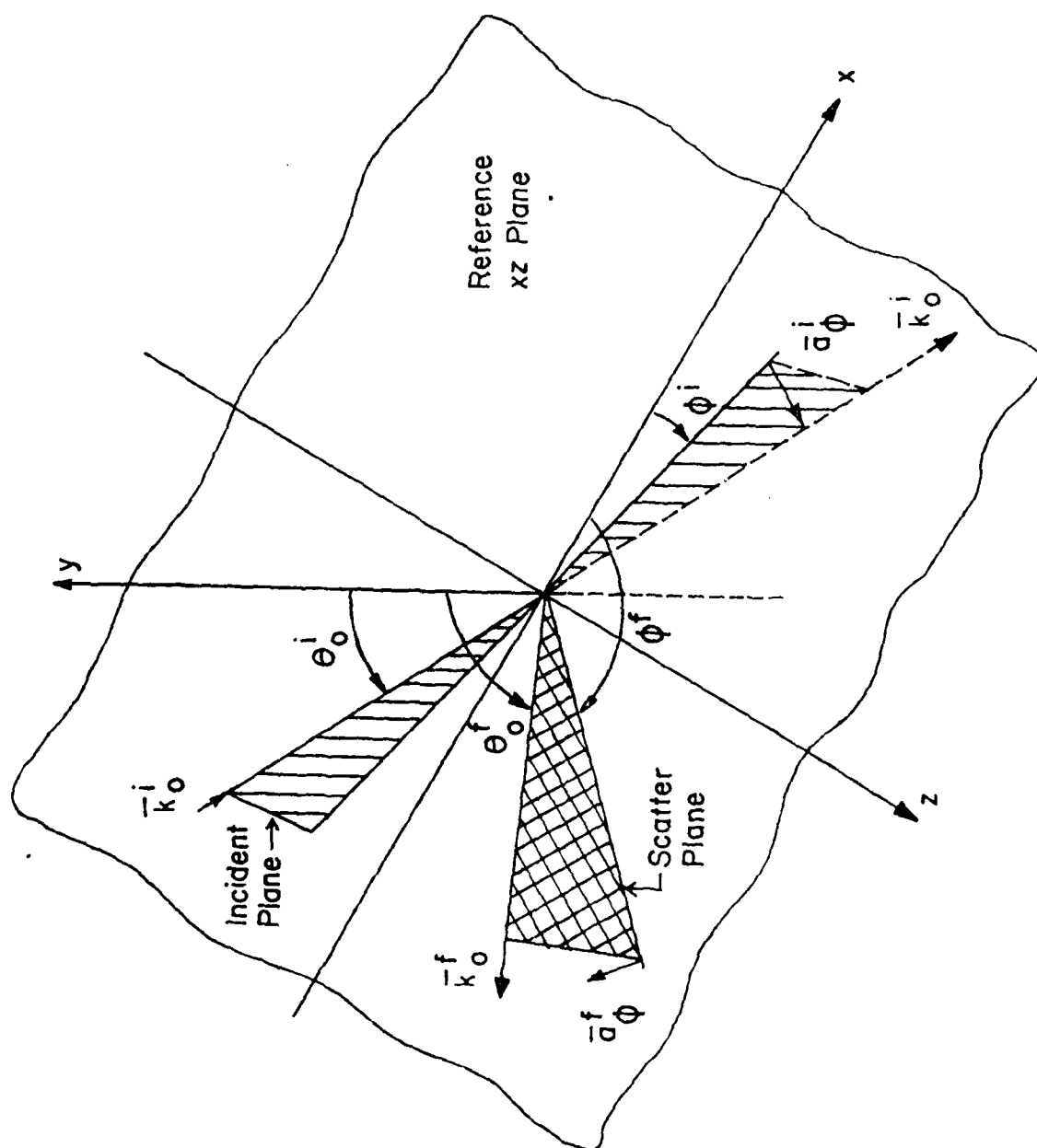


Fig. 1.



## PART II

## SCATTERING CROSS SECTIONS FOR COMPOSITE RANDOM SURFACES--

## FULL WAVE ANALYSIS

## Abstract

The full wave approach to rough surface scattering is applied to composite models of rough surfaces. In this work the principal distinguishing features of the individual rough surface is its correlation distance. Thus this model can be applied to scattering by rough seas as well as hilly terrain.

It is shown that the full wave approach accounts for both specular scatter and Bragg scattering. The scattering cross section for the composite surface, with two or more roughness scales, is shown to be a weighted sum of the scattering cross sections for the individual rough surface heights. Shadowing effects are accounted for explicitly in the analysis. The full wave solutions satisfy reciprocity, duality and realizability relationships in electromagnetic theory.

## 1. Introduction

Solutions have been derived for the scattering cross sections per unit area of the rough surface using the full wave approach (Bahar 1981a,b). For slightly rough surfaces the full wave solutions are shown to reduce to the perturbation solution (Barrick and Peake 1968, Barrick 1970). When the major contributions to the scattered fields come from the neighborhood of stationary phase (specular) points of the rough surface, the full wave solutions are shown to reduce to physical optics--solutions (Beckmann and Spizzichino, 1963). Since the results of the two general approaches to random rough scattering, perturbation and physical optics, are shown to be special cases of the full wave solutions, the limitations of these approaches can be precisely determined and the differences between them reconciled, (Bahar 1981a,b).

The principal motivations for this work are to extend the full wave analysis to composite rough surfaces with multiple roughness scales and to explicitly account for shadowing effects in the results. The main distinguishing feature of the individual rough surface  $h_i$  is its correlation distance  $\ell_i$ . However, no restrictions are made on the variance of the rough surface heights  $\sigma_{oi}$ . This work can therefore be applied to scattering by rough seas or by hilly terrain. In the treatment of composite rough surfaces by Brown (1978) the feature that distinguishes the two surfaces considered,  $h_1$  and  $h_2$ , is the surface wave number  $k_d$  where the "spectral splitting" occurs.

The principal expressions derived for the scattering cross section, using the full wave approach, are summarized in Section 2. The high frequency approximations are extended in Section 3, to cases in which the reference planes of incidence and scatter are not coplanar. In Section 4 a composite model of the rough surface with different roughness scales is analyzed. A two scale model is first considered and the result is given by equation (44). Using this model, the full wave analysis is shown to account for both specular scatter and Bragg scattering (54). For composite surfaces with  $N$  uncorrelated surface heights  $h_i$ , the solution is given by (55). In general, it is shown that the scattering cross section for the composite rough surface is a weighted sum of the cross sections for the individual rough surfaces  $h_i$ . In Section 5 shadowing effects are explicitly accounted for in the analysis.

## 2.1 Formulation of the Problem

The expression for the scattering cross section per unit area, based on the full wave solutions for the incoherent fields, is given by (Bahar 1981b)

$$\langle \sigma^{PQ} \rangle = \frac{k_0^2}{\pi} \int \langle S^{PQ} \cdot |\chi_2(v_y, -v_y) - |\chi(v_y)|^2 \exp[i v_x x_d + i v_z z_d] dx_d dz_d \rangle \quad (1)$$

The superscripts P, Q = V, H denote vertically and horizontally polarized waves respectively. The first superscript denotes the polarization of the scattered wave and the second the polarization of the incident wave. The symbol  $\langle \rangle$  denotes statistical average. Furthermore,

$$\langle S^{PQ} \rangle = \left\langle \frac{D^{PQ}(\vec{r}) D^{PQ*}(\vec{r}') U(\vec{r}) U(\vec{r}')}{(\vec{n} \cdot \vec{a}_y)(\vec{n}' \cdot \vec{a}_y)} \right\rangle + \left\langle \left| \frac{D^{PQ} U}{\vec{n} \cdot \vec{a}_y} \right|^2 \right\rangle \quad (2)$$

in which  $D^{PQ}(\vec{r})$  the scattering coefficients,  $U(\vec{r})$  is the shadow function and  $\vec{n}$  is the unit vector normal to the rough surface (Bahar 1981b). The vector  $\vec{v}$  is

$$\vec{v} = k_0 (\vec{n}^f - \vec{n}^i) = v_x \vec{a}_x + v_y \vec{a}_y + v_z \vec{a}_z \quad (3)$$

The free space wave number is  $k_0$  and the unit vectors in the direction of the incident and scattered waves are  $\vec{n}^i$  and  $\vec{n}^f$  respectively. The rough surface characteristic function and the joint characteristic function are  $\chi(v_y)$  and  $\chi_2(v_y, -v_y)$  respectively. It is assumed that rough surfaces are statistically homogeneous and isotropic, thus the surface height correlation function  $\langle h(x, z) h(x', z') \rangle$  depends only on the distances (see Fig. 1)

$$|\vec{r}_d| = (x-x')\vec{a}_x + (z-z')\vec{a}_z = x_d \vec{a}_x + z_d \vec{a}_z \quad (4)$$

and the correlation distance is smaller than the width of the illuminated surface.

Furthermore in (1) it is assumed that the rough surface height and slope are uncorrelated. It is also assumed that distribution of the shadow function  $U(\vec{r}_s)$

depends primarily upon the probability density function for the slope of the rough surface (Sancer 1969). The elements  $D^{PQ}$  (P, Q=V, H) of the 2 x 2 matrix D are given by (Bahar 1981a, b)

$$D = C_o^{in} T^f T^i = C_o^{in} \begin{bmatrix} C_\psi^f (F^{VV} C_\psi^i - F^{VH} S_\psi^i) - S_\psi^f (F^{HV} C_\psi^i - F^{HH} S_\psi^i) & C_\psi^f (F^{VV} S_\psi^i + F^{VH} C_\psi^i) - S_\psi^f (F^{HV} S_\psi^i + F^{HH} C_\psi^i) \\ S_\psi^f (F^{VV} C_\psi^i - F^{VH} S_\psi^i) + C_\psi^f (F^{HV} C_\psi^i - F^{HH} S_\psi^i) & S_\psi^f (F^{VV} S_\psi^i + F^{VH} C_\psi^i) + C_\psi^f (F^{HV} S_\psi^i + F^{HH} C_\psi^i) \end{bmatrix} \quad (5)$$

in which, for the incident and scattered waves (denoted by superscripts  $i$  and  $f$  respectively)

$$T^i = \begin{pmatrix} C_\psi^i & S_\psi^i \\ -S_\psi^i & C_\psi^i \end{pmatrix} \quad \text{and} \quad T^f = \begin{pmatrix} C_\psi^f & -S_\psi^f \\ S_\psi^f & C_\psi^f \end{pmatrix}, \quad \begin{matrix} C_\psi^i = \cos \psi^i, S_\psi^i = \sin \psi^i \\ C_\psi^f = \cos \psi^f, S_\psi^f = \sin \psi^f \end{matrix} \quad (6)$$

The transformation matrices  $T^i$  and  $T^f$  relate the vertically and horizontally polarized components with respect to the reference plane (normal to  $\bar{a}_y$ ) to the vertically and horizontally polarized components with respect to the local plane normal to the unit vector

$$\bar{n} = \bar{n}(x, h_z) = \nabla f / |\nabla f| = (-h_x \bar{a}_x + \bar{a}_y - h_z \bar{a}_z) / [h_x^2 + 1 + h_z^2]^{1/2} \quad (7a)$$

in which

$$h_x = \partial h / \partial x \quad \text{and} \quad h_z = \partial h / \partial z \quad (7b)$$

The function

$$f(x, y, z) = y - h(x, z) = 0 \quad (8)$$

defines the rough surface between medium 0,  $y > h(x, z)$ , and medium 1,  $y < h(x, z)$ . The angle between the reference and local planes of incidence and scattering are  $\psi^i$  and  $\psi^f$  respectively. The elements of the matrix

$$F = \begin{pmatrix} F^{VV} & F^{VH} \\ F^{HV} & F^{HH} \end{pmatrix} \quad (9)$$

are functions of the angles of incidence and scatter in the local coordinate system and the relative permittivity  $\epsilon_r$  and permeability  $\mu_r$  of the two media surrounding the interface  $f(x, y, z) = 0$ . Thus  $F^{PQ}$  ( $P=V, H$ ) are explicitly functions of the unit vectors (see Bahar 1981a, b)

$$\bar{n}^i = \sin\theta_o^i \cos\phi^i \bar{a}_x - \cos\theta_o^i \bar{a}_y + \sin\theta_o^i \sin\phi^i \bar{a}_z \quad (10a)$$

$$\bar{n}^f = \sin\theta_o^f \cos\phi^f \bar{a}_x + \cos\theta_o^f \bar{a}_y + \sin\theta_o^f \sin\phi^f \bar{a}_z \quad (10b)$$

$$\bar{n} = \sin\gamma \cos\delta \bar{a}_x + \cos\gamma \bar{a}_y + \sin\gamma \sin\delta \bar{a}_z \quad (11)$$

and the electromagnetic parameters  $\epsilon_r$  and  $\mu_r$  respectively (Bahar 1981b).

It has been shown (Bahar 1981b), that on assuming

$$k_o^2 < h^2 > = k_o^2 \sigma_o^2 \ll 1, \bar{n} \cdot \bar{a}_y \approx 1 \text{ and } U(\bar{r}_s) \approx 1 \quad (12)$$

the full wave solutions for the scattering cross sections per unit area reduce to the first order perturbation solutions (Barrick and Peake 1968, Barrick 1970)

$$< \sigma_o^{PQ} > = 4\pi k_o^4 W(v_x, v_z) |\cos\theta_o^f \cos\theta_o^i \alpha^{PQ}|^2 \quad (13)$$

in which

$$2 \cos\theta_o^f \cos\theta_o^i \alpha^{PQ} = \pm [C_o^{in,PQ} (\cos\theta_o^f + \cos\theta_o^i)]_{\bar{n} \rightarrow \bar{a}_y} \quad (14a)$$

The upper and lower signs in (14a) are used for Q=H and V respectively. The spectral density of the rough surface height function is given by the Fourier transform of the height correlation function (Barrick 1970, Ishimaru 1978).

$$W(v_x, v_z) = \frac{1}{\pi^2} \int_{-\infty}^{\infty} \exp[iv_x x_d + iv_z z_d] < h(x, z) h(x', z') > dx_d dz_d \quad (14b)$$

Subject to the perturbation approximation, the local angles of incidence and scatter are approximated by the angles of incidence and scatter with respect to the reference plane normal to  $\bar{a}_y$ . Thus for instance

$$C_o^{in} = \cos\theta_o^{in} = -\bar{n}^i \cdot \bar{n} \approx -\bar{n}^i \cdot \bar{a}_y = \cos\theta_o^i = C_o^i \quad (15)$$

Under special conditions (Bahar 1980b), the high frequency approximation of the full wave solution (1) is obtained by setting

$$\bar{n} = \bar{n}_s, [\bar{n}^i \bar{a}_y \bar{n}_s] = [\bar{n}^f \bar{a}_y \bar{n}_s] = [\bar{n}^i \bar{a}_y \bar{n}^f] = 0 \text{ and } U(\bar{r}_s) = 1 \quad (16)$$

Thus, if the planes of incidence and scatter with respect to the reference plane ( $y=0$ ) and the local plane at the specular points (where  $\bar{n} = \bar{n}_s$ ) are coplanar, and if the specular points exist on the rough surface,

$$\langle \sigma_{\infty}^{PQ} \rangle = \frac{k_o^2}{\pi} \left( \frac{2 \cos^2 \theta_o^s}{\cos \theta_o^f + \cos \theta_o^i} \right)^2 |R_{10}^P(\theta_o^s)|^2 \mathcal{J} \delta_{PQ} \quad (17a)$$

in which  $\mathcal{J}$  is given by

$$\mathcal{J} = \int_{-\infty}^{\infty} [\chi_2(v_y, -v_y) - |\chi(v_y)|^2] \exp[iv_x x_d + iv_z z_d] dx_d dz_d \quad (17b)$$

and  $R_{10}^P(\theta_o^s)$  is the Fresnel reflection coefficient for vertically and horizontally polarized waves (P=V,H) evaluated at stationary phase points where

$$\bar{n} = \bar{n}_s = (\bar{n}^f - \bar{n}^i) / |\bar{n}^f - \bar{n}^i| \quad (18)$$

Thus

$$(\bar{n}^i \cdot \bar{n})^2 = (\bar{n}^f \cdot \bar{n})^2 = \cos^2 \theta_o^s = (C_o^s)^2 = \frac{1}{2} [1 + \cos \theta_o^i \cos \theta_o^f - \sin \phi_o^i \sin \phi_o^f \cos(\phi^f - \phi^i)] \quad (19)$$

With the exception of the reflection coefficient  $R_{10}^P(\theta_o^s)$ , which is evaluated at the stationary phase (specular) point rather than at the angle of incidence  $\theta_o^i$ , the solution (17) is the same as the Physical Optics solution (Barrick 1970, Beckman and Spizzichino 1963, Ishimaru 1978). Thus for very good conducting boundaries ( $|R_{10}^P(\theta_o^s)| \rightarrow 1$ ),  $\langle \sigma_{\infty}^{PP} \rangle$  is independent of polarization in the Physical Optics limit. Furthermore in view of the Kronecker delta  $\delta_{PQ}$  in (17) there is no depolarization ( $\langle \sigma_{\infty}^{PQ} \rangle = 0$  for  $P \neq Q$ ). This is due to the assumption  $[\bar{n}^i \cdot \bar{a}_y \cdot \bar{n}^f] = 0$  (16).

Using the full wave solution (1) as a starting point for the present analysis, the scattering cross sections are derived for cases in which neither the assumptions made for the perturbation analysis (12) nor the assumptions made for the Physical Optics analysis (16) are valid. For instance, even for very high frequencies, the assumption that the major contributions to the (incoherent) scattering cross section come from the stationary phase (specular) points of the rough surface cannot in general be satisfied (Bahar 1981b).

In view of experimental evidence that neither the perturbation nor the Physical Optics approach aptly determine the scattering cross section for all  $\bar{n}^i$  and  $\bar{n}^f$ , and the realization that simple, single scale models of rough surfaces are not suitable for a large variety of relevant problems, expressions for the scattering cross

sections have been derived for composite rough surfaces (Barrick and Peake 1968, Wright 1968, Valenzuela 1978, Brown 1978, Burrows 1973). The full wave approach will therefore also be applied to composite rough surfaces and compared with earlier approaches to this problem.

### 3. High Frequency Approximation When the Reference

Planes of Incidence and Scatter Are Not Coplanar.

The Physical Optics solution (17) was derived on assuming that the principal contributions to the incoherent scattered field come from the neighborhood of stationary phase (specular) points of the rough surface where  $\bar{n} \approx \bar{n}_s$ . Furthermore it is assumed that the phase in the integrand of the full wave solutions varies very rapidly for the shadow region and therefore  $U(\bar{r}_s)$  can be set equal to unity (Bahar 1981b). These assumptions are satisfied if the angle  $\frac{\pi}{2} - \theta_0^i$  is larger than the mean value of the slope of the rough surface  $\beta_0$  (Bahar 1981b). Thus to satisfy these assumptions it is necessary that

$$\tan\left(\frac{\pi}{2} - \theta_0^{i,f}\right) = \cot\theta_0^{i,f} > \tan\beta_0 = 2\sigma/\ell \quad (20)$$

where  $\sigma$  and  $\ell$  are the variance and the correlation distance for the rough surface. However, the additional assumption that the reference planes of incidence and scatter are coplanar

$$[\bar{n}^i \bar{a}_y \bar{n}^f] = 0 \quad (21)$$

is not satisfied in general even when (20) is satisfied. While the assumption (21) simplifies the Physical Optics solution (17) considerably, it results in no depolarization. In this section the restriction (21) is lifted and the appropriate high frequency results are derived.

The value of the matrix  $D$  at the stationary phase points is obtained by setting  $\bar{n} = \bar{n}_s$  (18) in equation (5). Thus

$$\begin{aligned}
 \left[ \frac{D}{\bar{n} \cdot \bar{a}} \right]_{\bar{n} \cdot \bar{n}_s} &= C_o^i F_3(\bar{n}^f, \bar{n}^i) \begin{bmatrix} C_\psi^{fs} R_{10}^{Vs} C_\psi^{is} + S_\psi^{fs} R_{10}^{Hs} S_\psi^{is} & C_\psi^{fs} R_{10}^{Vs} S_\psi^{is} - S_\psi^{fs} R_{10}^{Hs} C_\psi^{is} \\ S_\psi^{fs} R_{10}^{Vs} C_\psi^{is} - C_\psi^{fs} R_{10}^{Hs} S_\psi^{is} & S_\psi^{fs} R_{10}^{Vs} S_\psi^{is} + C_\psi^{fs} R_{10}^{Hs} C_\psi^{is} \end{bmatrix} \\
 &= C_o^i C_\psi^{fs} C_\psi^{is} F_3(\bar{n}^f, \bar{n}^i) \begin{bmatrix} R_{10}^{Vs} + R_{10}^{Hs} \tan \psi^{fs} \tan \psi^{is} & R_{10}^{Vs} \tan \psi^{is} - R_{10}^{Hs} \tan \psi^{fs} \\ R_{10}^{Vs} \tan \psi^{fs} - R_{10}^{Hs} \tan \psi^{is} & R_{10}^{Vs} \tan \psi^{fs} \tan \psi^{is} + R_{10}^{Hs} \end{bmatrix}
 \end{aligned} \quad (22)$$

in which

$$C_o^i F_3(\bar{n}^f, \bar{n}^i) = 2 \cos^2 \theta_o^s / (C_o^f + C_o^i), \quad R_{10}^{Ps} = R_{10}^P(\theta_o^s) \quad (23)$$

$$C_\psi^{is} = \cos \psi^{is} = \frac{C_o^i + C_o^f - C_o^i (1 + \cos 2\theta_o^s)}{S_o^i \sin 2\theta_o^s}, \quad S_\psi^{is} = \sin \psi^{is} = \frac{S_o^f \sin(\phi^f - \phi^i)}{\sin 2\theta_o^s} \quad (24a)$$

and

$$C_\psi^{fs} = \cos \psi^{fs} = \frac{C_o^i + C_o^f - C_o^f (1 + \cos 2\theta_o^s)}{S_o^f \sin 2\theta_o^s}, \quad S_\psi^{fs} = \sin \psi^{fs} = \frac{S_o^i \sin(\phi^f - \phi^i)}{\sin 2\theta_o^s} \quad (24b)$$

The angle  $\theta_o^s$  is given by (19) and

$$C_o^i = \cos \theta_o^i, \quad S_o^i = \sin \theta_o^i, \quad C_o^f = \cos \theta_o^f, \quad S_o^f = \sin \theta_o^f \quad (25)$$

Thus the stationary phase, high frequency approximation, for the scattering cross sections for  $P = V, H$  and  $Q = H, V$  ( $P \neq Q$ ) are

$$\langle \sigma_\infty^{PP} \rangle = \frac{k_o^2}{\pi} \left( \frac{2 C_\psi^{fs} C_\psi^{is} \cos^2 \theta_o^s}{C_o^f + C_o^i} \right)^2 |R_{10}^{Ps} + R_{10}^{Qs} \tan \psi^{fs} \tan \psi^{is}|^2 \quad (26a)$$

and

$$\langle \sigma_\infty^{PQ} \rangle = \frac{k_o^2}{\pi} \left( \frac{2 C_\psi^{fs} C_\psi^{is} \cos^2 \theta_o^s}{C_o^f + C_o^i} \right)^2 |R_{10}^{Ps} \tan \psi^{is} - R_{10}^{Qs} \tan \psi^{fs}|^2 \quad (26b)$$

For highly conducting boundaries

$$\langle \sigma_\infty^{VV} \rangle = \langle \sigma_\infty^{HH} \rangle \text{ and } \langle \sigma_\infty^{VH} \rangle = \langle \sigma_\infty^{HV} \rangle \quad (27)$$

For backscatter however, since

$$S_\psi^{is} = S_\psi^{fs} = 0 \text{ and } \cos \theta_o^s = 1 \quad (28)$$



both (17) and (26) reduce to

$$\langle \sigma_{\infty}^{PP} \rangle_B = \frac{1}{\pi} \left( \frac{k_0}{C_0} \right)^2 \left| R_{10}^{Ps} \right|^2 \mathcal{J} \quad (29a)$$

and

$$\langle \sigma_{\infty}^{PQ} \rangle_B = 0, \quad P \neq Q \quad (29b)$$

For slightly rough surfaces (Bahar 1981b)

$$\mathcal{J} = \pi^2 k_0^2 (C_0^f + C_0^i)^2 W(v_x, v_z) \quad (30)$$

Thus, except for normal incidence the perturbation and stationary phase approximations for  $\langle \sigma^{PQ} \rangle_B$  for slightly rough surfaces are not in agreement. However it should be noted that for backscatter both the small slope perturbation solution  $\bar{n} \approx \bar{a}_y$  and stationary phase approximations for  $\langle \sigma^{PQ} \rangle_B$  ( $P \neq Q$ ) vanish. The full wave solution for  $\langle \sigma^{PQ} \rangle_B$  ( $P \neq Q$ ) does not vanish since  $S^{PQ}$  (1) vanishes for backscatter only at the specular points or where  $\bar{n} \approx \bar{a}_y$ .

#### 4. Composite Surface with Multiple Roughness Scales

In this section it is assumed that the composite surface under consideration has two or more distinct classes of roughness that are uncorrelated. Consider first the case in which the two statistically independent surface height variances  $\sigma_{o1}$  and  $\sigma_{o2}$  are small and

$$(v_y \sigma_{o1})^2 \ll 1 \text{ and } (v_y \sigma_{o2})^2 \ll 1, \quad \langle h_1 \rangle = 0, \langle h_2 \rangle = 0 \quad (31a)$$

$$\bar{n} \approx \bar{a}_y \quad (31b)$$

Thus  $\langle S^{PQ} \rangle \rightarrow [S^{PQ}]_{\bar{a}_y}$  can be factored out of the integral (1) and since

$$\exp[i v_y (h_1 + h_2)] \sim 1 + i v_y (h_1 + h_2) - \frac{v_y^2}{2} (h_1 + h_2)^2 \quad (32)$$

it follows that

$$\chi(v_y) = 1 - \frac{v_y^2}{2} (\langle h_1^2 \rangle + \langle h_2^2 \rangle) / 2 = 1 - \frac{v_y^2}{2} (\sigma_{o1}^2 + \sigma_{o2}^2) / 2 \quad (33)$$

and

$$\begin{aligned} \chi_2(v_y, -v_y) &= 1 - \frac{v_y^2}{2} (\langle h_1^2 \rangle + \langle h_2^2 \rangle - \langle h_1 h_1' \rangle - \langle h_2 h_2' \rangle) \\ &= 1 - \frac{v_y^2}{2} [\sigma_{o1}^2 (1 - C_1) - \sigma_{o2}^2 (1 - C_2)] \end{aligned} \quad (34)$$

in which the surface height correlation functions  $\langle h_1 h_1' \rangle$  and  $\langle h_2 h_2' \rangle$  are expressed in terms of the correlation coefficients  $C_1$  and  $C_2$  that depend only upon distance  $\bar{r}_d$  (4). Thus the integral  $\mathcal{J}$  (17b) reduces to

$$\mathcal{J} = \int_{-\infty}^{\infty} \exp[iv_x x_d + iv_z z_d] v_y^2 (\langle h_1 h_1' \rangle + \langle h_2 h_2' \rangle) dx_d dz_d$$

$$= \mathcal{J}_1 + \mathcal{J}_2 = \pi^2 v_y^2 [W_1(v_x, v_z) + W_2(v_x, v_z)] \quad (35a)$$

in which for  $i = 1, 2$

$$\mathcal{J}_i = \int_{-\infty}^{\infty} \exp[iv_x x_d + iv_z z_d] [X_2^i - |X^i|^2] dx_d dz_d \quad (35b)$$

The characteristic functions and the spectral density functions for the rough surface height  $h_i$  are  $X_2^i$  and  $W_i$  respectively. Thus, in this case since in (1),  $\bar{n} \approx \bar{n}_y$ , the scattering cross sections for the incoherent fields is the sum of the scattering cross sections for the individual slightly rough surfaces  $h_1$  and  $h_2$ .

For the more general case the surface height variances  $\sigma_{o1}$  and  $\sigma_{o2}$  are not assumed to be small. However, the correlation distance  $\ell_1$  is assumed to be much smaller than  $\ell_2$ . Thus by definition

$$C_1(\rho=\ell_1) = C_1/e \quad \text{and} \quad C_2(\rho=\ell_2) = C_2/e \quad (36)$$

in which  $\rho = (x_d^2 + z_d^2)^{1/2}$ ,  $e$  is the Neperian number and it is assumed that,

$$\ell_1 \ll \ell_{12} \ll \ell_2 \quad (37)$$

where  $\ell_{12}$  is a constant. In this case  $\langle \sigma^{PQ} \rangle$  (1) is given by

$$\langle \sigma^{PQ} \rangle = \frac{k^2}{\pi} \int \langle \sigma^{PQ} \rangle [X_2^1 X_2^2 - |X^1 X^2|^2] \exp[iv_x x_d + iv_z z_d] dx_d dz_d \quad (38)$$

The significance of the assumption (37) is that for distances  $0 < \rho < \ell_{12}$

$$X_2^1(0) = 1 > X_2^1 > |X^1|^2 \quad \text{and} \quad X_2^2 \approx X_2^2(0) = 1 \quad (39)$$

and for distance  $\rho > \ell_{12}$

$$\chi_2^1 \rightarrow |\chi^1|^2 \text{ and } \chi_2^2(0) = 1 > \chi_2^2 > |\chi^2|^2 \quad (40)$$

Assuming for example normal distributions for  $h_i$

$$\chi^i(\mathbf{v}_y) = \exp(-\mathbf{v}_y^2 \sigma_{oi}^2 / 2) \quad , \quad i=1,2 \quad (41)$$

and

$$\chi_2^i(\mathbf{v}_y, -\mathbf{v}_y) = \exp[-\mathbf{v}_y^2 \sigma_{oi}^2 (1-C_i(\rho))] \quad i=1,2 \quad (42)$$

Thus

$$\begin{aligned} \chi_2^1 \chi_2^2 - |\chi^1 \chi^2|^2 &= \exp(-\mathbf{v}_y^2 \sigma_{o1}^2) \left\{ \exp[\mathbf{v}_y^2 \sigma_{o1}^2 C_1 - \mathbf{v}_y^2 \sigma_{o2}^2 (1-C_2)] - \exp(-\mathbf{v}_y^2 \sigma_{o2}^2) \right\} \\ &= \exp(-\mathbf{v}_y^2 \sigma_{o1}^2) \left\{ \exp(\mathbf{v}_y^2 \sigma_{o1}^2 C_1) - 1 + \exp[-\mathbf{v}_y^2 \sigma_{o2}^2 (1-C_2)] - \exp(-\mathbf{v}_y^2 \sigma_{o2}^2) \right\} \\ &= \chi_2^1 - |\chi^1|^2 + |\chi^1|^2 [\chi_2^2 - |\chi^2|^2] \end{aligned} \quad (43)$$

and the scattering cross section can be written as

$$\langle \sigma^{PQ} \rangle = \langle \sigma^{PQ} \rangle_1 + |\chi^1|^2 \langle \sigma^{PQ} \rangle_2 \quad (44a)$$

in which for  $i = 1, 2$ ,

$$\langle \sigma^{PQ} \rangle_i = \frac{k_o^2}{\pi} \int \langle S^{PQ} \rangle [ \chi_2^i - |\chi^i|^2 ] \exp[i\mathbf{v}_x \mathbf{x}_d + i\mathbf{v}_z \mathbf{z}_d] d\mathbf{x}_d d\mathbf{z}_d \quad (44b)$$

and  $\langle S^{PQ} \rangle$ , the statistical average of  $S^{PQ}$ , (2) is given by

$$\int S^{PQ} p(\bar{\mathbf{n}}, U) d\mathbf{h}_x d\mathbf{h}_z dU \quad (44c)$$

where  $p(\bar{\mathbf{n}}, U)$  is the joint probability density function (Sancer 1969). If the variance of the surface height  $h_1$  is small,  $|\chi^1| \approx 1$ , and (43) reduces to

$$\langle \sigma^{PQ} \rangle = \langle \sigma^{PQ} \rangle_1 + \langle \sigma^{PQ} \rangle_2, \quad \mathbf{v}_y^2 \sigma_{o1}^2 \ll 1 \quad (45)$$

Before the transform in (44b) can be evaluated it is necessary to determine  $\langle S^{PQ} \rangle$ ,

(44c) for the composite rough surface. When the stationary phase approximations are valid for high frequencies and shadowing effects are not very significant  $\bar{\mathbf{n}}$  can be replaced by  $\bar{\mathbf{n}}_s$  in (45) and as in (26)

$$\langle S^{PQ} \rangle \approx S_\infty^{PQ} = \left| \frac{D^{PQ}}{\bar{\mathbf{n}} \cdot \bar{\mathbf{a}}_y} \right|_{\bar{\mathbf{n}} \rightarrow \bar{\mathbf{n}}_s}^2 \quad (46)$$

When the mean value of the slope of the composite surface,  $\beta_o$  (20) is very small

$$\tan \beta_o = 2\sigma/\ell \ll 1, \quad \bar{\mathbf{n}} \approx \bar{\mathbf{a}}_y \quad (47)$$

the perturbation approximation for (44c) can be used as in (13)

$$\langle S^{PQ} \rangle \approx S_o^{PQ} = \left| D^{PQ} \right|_{\vec{n} \cdot \vec{a}_y}^2 \quad (48)$$

For near grazing angles where

$$\cot \theta_o^{i,f} \ll \tan \beta_o = 2\sigma/\ell \quad (49)$$

shadowing effects become significant and even at high frequencies (46) cannot be used. In this case, since  $U(\vec{r}_s) = 1$  only for those portions of the rough surface that are almost horizontal ( $\vec{n} \cdot \vec{a}_y \approx 1$ ), the perturbation approximation  $\vec{n} \rightarrow \vec{a}_y$  could be used for near grazing angles (49) even at high frequencies.

For the general case however, it is necessary to evaluate  $\langle S^{PQ} \rangle$  (44c), using the statistics of the slopes of the composite surface (see Sec. 5). Provided that (37) is satisfied and either the perturbation or stationary phase approximations are valid such that  $S^{PQ}$  is independent of position, (46), (48), the scattering cross sections (1) can be expressed as follows:

$$\langle \sigma^{PQ} \rangle = \langle \sigma^{PQ} \rangle_1 + |X|^2 \langle \sigma^{PQ} \rangle_2 = \frac{k_o^2}{\pi} \left[ \langle S^{PQ} \rangle_1 \mathcal{J}_1 + |X|^2 \langle S^{PQ} \rangle_2 \mathcal{J}_2 \right] \quad (50)$$

For rough surfaces with normal distributions (Beckmann and Spizzichino 1963)

$$\mathcal{J}_i = \begin{cases} \pi (v_{y\sigma_{oi}} \ell_i)^2 \exp(-v_{y\sigma_{oi}}^2 - v_{xz}^2 \ell_i^2/4) = \pi^2 v_y^2 W(v_x, v_z), & (v_{y\sigma_{oi}})^2 \ll 1 \\ \pi \ell_i^2 \exp(-v_{y\sigma_{oi}}^2) \sum_{m=1}^{\infty} \frac{(v_{y\sigma_{oi}})^{2m}}{m!m} \exp(-v_{xz}^2 \ell_i^2/4), & v_{y\sigma_{oi}} \approx 1 \\ \frac{\pi \ell_i^2}{v_{y\sigma_{oi}}^2} \exp[-(v_{xz} \ell_i/2 v_{y\sigma_{oi}})^2] & , (v_{y\sigma_{oi}})^2 \gg 1 \end{cases} \quad (51)$$

where

$$v_{xz} = (v_x^2 + v_z^2)^{1/2} \quad (52)$$

The expression (50) simplifies considerably if the variance of the surface height  $h_1$  is small (44) while the variance of the surface height  $h_2$  is large ( $v_{y\sigma_{oi}}^2 \ll 1$ )

and  $v_{y02}^2 \gg 1$ ). In this special case the contribution from the term  $\langle \sigma^{PQ} \rangle_2$  is dominant when

$$\cot \alpha_{01}^{i,f} \geq \tan \beta_0 = 2\sigma/l \quad (53)$$

On the other hand only horizontal portions of the rough surface ( $\bar{n} = \bar{a}_y$ ) are both illuminated and visible for grazing angles, (49), and the contribution from the term  $\langle \sigma^{PQ} \rangle_1$  is dominant near grazing angles. Thus if in addition to condition (37)  $v_{y02}^2 \ll 1$  and  $v_{y02}^2 \gg 1$ , at high frequencies (50) can be approximated by

$$\langle \sigma^{PQ} \rangle = \frac{k_0^2}{\pi} [S_o^{PQ} \langle \mathcal{A}_1 \rangle + S_\infty^{PQ} \langle \mathcal{A}_2 \rangle] \equiv \langle \sigma^{PQ} \rangle_1 + \langle \sigma^{PQ} \rangle_2 \quad (54)$$

in which  $S_o^{PQ}$  and  $S_\infty^{PQ}$  are given by (46) and (48) respectively. The scattering cross sections for such composite surfaces are approximately equal to the sum of the individual cross sections  $\langle \sigma^{PQ} \rangle_1$  and  $\langle \sigma^{PQ} \rangle_2$  for the rough surface heights  $h_1$  and  $h_2$  respectively (Barrick and Peake 1968, Barrick 1970).

The result (43) can be generalized for a composite surface represented by the superposition of  $N$  uncorrelated surface heights  $h_n$  ( $n = 1, 2, \dots, N$ ) for which the correlation distances  $l_n$  satisfy

$$l_1 \ll l_{12} \ll l_2 \ll l_{23} \ll l_3 \dots l_{N-1} \ll l_{N-1,N} \ll l_N \quad (55)$$

In this case

$$\langle \sigma^{PQ} \rangle = \sum_{n=1}^N \prod_{m=1}^n |X^{m-1}|^2 \langle \sigma^{PQ} \rangle_n = \sum_{n=1}^N w_n \langle \sigma^{PQ} \rangle_n \quad (56a)$$

where

$$\langle \sigma^{PQ} \rangle_n = \frac{k_0^2}{\pi} \int_{-\infty}^{\infty} \langle S^{PQ} \rangle [X_2^n - |X^n|^2] \exp[iv_x x_d + iv_z z_d] dx_d dz_d \quad (56b)$$

The characteristic functions for the surface height  $h_n$  are  $X_2^n(v_y, -v_y)$  and  $X^n(v_y)$ , and  $X^0 \equiv 1$ . In (55) the contribution  $\langle \sigma^{PQ} \rangle_n$  to the total scattering cross section,  $\langle \sigma^{PQ} \rangle$ , due to the rough surface  $h_n$  is weighted by the product

$$w_n = \prod_{m=1}^n |X^{m-1}|^2 \quad (57)$$

Since  $|X^1| \leq 1$  (40), the weighting factor  $w_n$  is in general less than unity.

It decreases monotonically as  $n$  increases even when the variance  $\sigma_{oi}$  of the individual rough surfaces are small ( $v_{yoi}^2 \ll 1$ ,  $i = 1, 2, \dots, N$ ).

The effects of shadowing have not been accounted for explicitly in the solution (54). To do so it is necessary to determine the statistical averages  $\langle S^{PQ} \rangle$  (45) using the joint probability density function  $p(\bar{n}, U)$  for the rough surface slopes  $\nabla f$  (7) and the shadowing function  $U(\bar{r}_s)$ . Both terms of the product

$$\langle S^{PQ} \rangle [\chi_0^n - |\chi^n|^2] = K(x_d, z_d) \quad (58)$$

are functions of distance  $\bar{r}_d$  (4). The two dimensional Fourier transform of the product  $K(x_d, z_d)$ , (56) can be evaluated by determining the two dimensional convolutions of the two dimensional Fourier transforms for  $\langle S^{PQ} \rangle$  and  $[\chi_0^n - |\chi^n|^2]$  (Brown 1978, 1980). However for high frequencies the integral expression for the scattered fields (Bahar 1981b) may also be evaluated at the stationary phase points before squaring and averaging (Kodis 1966). Barrick (1968) shows that the results are the same regardless of the order of evaluating the integral and the statistical average. This latter approach which simplifies the analysis will be used in Section 5 to explicitly account for shadowing in the evaluation of  $\langle \sigma_n^{PQ} \rangle$  (56).

### 5. Scattering Cross Sections for Composite Rough Surfaces

#### When Shadowing Effects are Accounted for Explicitly

To evaluate  $\langle S^{PQ} \rangle$  (44c) for high frequencies it is convenient to represent the joint probability density function  $p(\bar{n}, U)$  in terms of the conditional density  $P(U|\bar{n})$  (Sancer 1969, Brown 1978, 1980)

$$p(\bar{n}, U) = p(\bar{n})P(U|\bar{n}) \quad (59)$$

in which  $p(\bar{n})$  is the density function of the gradient of the rough surface  $\nabla f = \bar{n}|\nabla f|$  (7) and,

$$P(U|\bar{n}) = P_2(\bar{n}^f, \bar{n}^i|\bar{n})\delta(U-1) + [1-P_2(\bar{n}^f, \bar{n}^i|\bar{n})]\delta(U) \quad (60)$$

The Dirac delta function is  $\delta(U)$  and  $P_2(\bar{n}^f, \bar{n}^i | \bar{n})$  is the probability that a point will be both illuminated by a source at  $-r_o \bar{n}^i$  and visible at the observation point  $r \bar{n}^f$ , given the value of the gradient  $\bar{n}(h_x, h_z)$ . Thus for high frequencies  $\langle S^{PQ} \rangle$  can be replaced by its stationary phase approximation (Sancer 1969)

$$\begin{aligned} \langle S_{\infty}^{PQ} \rangle &= \int \left| \frac{D^{PQ}}{\bar{n} \cdot \bar{a}_y} \right|_{\bar{n}_s}^2 p(\bar{n}) p(U | \bar{n}_s) U^2(\bar{r}_s) dh_x dh_z dU \\ &= \left| \frac{D^{PQ}}{\bar{n} \cdot \bar{a}_y} \right|_{\bar{n}_s}^2 P_2(\bar{n}^f, \bar{n}^i | \bar{n}_s) \end{aligned} \quad (61)$$

The expression for the probability  $P_2(\bar{n}^f, \bar{n}^i | \bar{n}_s)$  has been given by Sancer (1969) for rough surfaces with normal distributions. The above approximation for  $\langle S^{PQ} \rangle$  is appropriate for scattering from the rough surface  $h_2$  with the large correlation distance  $\ell_2$ . Thus

$$\langle S^{PQ} \rangle_2 = \frac{k_o^2}{\pi} \left| \frac{D^{PQ}}{\bar{n} \cdot \bar{a}_y} \right|_{\bar{n}_s}^2 P_2(\bar{n}^f, \bar{n}^i | \bar{n}_s) \mathcal{J}_2 \quad (62)$$

in which  $\left| \frac{D^{PQ}}{\bar{n} \cdot \bar{a}_y} \right|_{\bar{n}_s}^2$  is given by (22) and  $\mathcal{J}_2$  (35b) is given by (51) for surfaces with normal distributions. Thus for surfaces with  $(v_y \sigma_{o2})^2 \gg 1$ ,  $\mathcal{J}_2$  is proportional to the joint probability density function  $p(\bar{n}_s)$  for the gradient at the stationary phase points (51).

For angles  $\theta_o^{i,f}$  smaller than the mean value of the slope  $\beta_o$  (53),  $\langle S^{PQ} \rangle_2$  is the dominant term in the solution (44). However for angles larger than  $\beta_o$  (near grazing angles  $\theta_o^{i,f}$ ) (49), scattering due to the small scale roughness dominates. This is either because the joint probability density function at the stationary phase points,  $p(\bar{n}_s)$  is very small or because the probability is very small that these stationary phase points are illuminated and visible,

$(P_2(\vec{n}^f, \vec{n}^i | \vec{n}_s) \ll 1)$ . Scattering due to the small scale roughness is most significant for near grazing angles where the contributions from the specular points are very small. Thus the contribution to the total scattering cross section due to the term  $\langle S_o^{PQ} \rangle_1$  (44) is primarily from the near horizontal portions of the rough surface that are both illuminated and visible. Hence for the term  $\langle S_o^{PQ} \rangle_1$ ,  $\langle S^{PQ} \rangle$  can be approximated by

$$\begin{aligned} \langle S_o^{PQ} \rangle &= \int \left| \frac{D^{PQ}}{\vec{n} \cdot \vec{a}_y} \right|_{\vec{n}=\vec{a}_y}^2 p(\vec{n}) p(U|\vec{n}) U^2 (\vec{r}_s) d\vec{h}_x d\vec{h}_z dU \\ &= \left| \frac{D^{PQ}}{\vec{n} \cdot \vec{a}_y} \right|_{\vec{n}=\vec{a}_y}^2 P_1(\vec{n}^f, \vec{n}^i, U=1) \end{aligned} \quad (63)$$

where

$$P_1(\vec{n}^f, \vec{n}^i, U=1) = \int P_2(\vec{n}^f, \vec{n}^i | \vec{n}) p(\vec{n}) d\vec{h}_x d\vec{h}_y \quad (64)$$

is the probability that the rough surface is both illuminated and visible.

Thus for the surface with the small roughness scale

$$\langle S_o^{PQ} \rangle_1 = \pi k_o^2 v_y^2 \left| \frac{D^{PQ}}{\vec{n} \cdot \vec{a}_y} \right|_{\vec{n}=\vec{a}_y}^2 P_1(\vec{n}^f, \vec{n}^i, U=1) W_1(v_x, v_z) \quad (65)$$

in which  $W_1(v_x, v_z)$  is the spectral density function for the rough surface  $h_1$  (35).

The scattering cross section for the composite rough surface is therefore obtained by summing (65) and (62) while accounting for the weighting function  $w_1 = |y|^2$  in (56). This weighting function ( $w_1 < 1$ ) has the effect of damping the contribution from the specular point because of the superposition of the surface  $h_1$  (with the small roughness scale) on the surface  $h_2$  (with the large roughness scale).

On deriving the expression (1) for the scattering cross section an integral containing the term

$$\Delta = \left\{ \left\langle \left| \frac{D^{PQU}}{\vec{n} \cdot \vec{a}_y} \right|^2 \right\rangle - \left| \left\langle \frac{D^{PQU}}{\vec{n} \cdot \vec{a}_y} \right\rangle \right|^2 \right\} |y|^2$$

has been neglected (Bahar 1981b). This term can be ignored either if  $|x|^2 \ll 1$  or when the perturbation or stationary phase approximations are valid.



## 6. Concluding Remarks

Using the full wave approach, a general expression has been derived for the scattering cross sections for composite rough surfaces comprised of a superposition of  $N$  uncorrelated rough surface heights  $h_i$ . The distinguishing feature of the different rough surface heights is the correlation distance  $\ell_i$  (55) and not the surface height variance. It is shown that the scattering cross sections for the composite surface are a weighted sum of the scattering cross sections for the individual rough surfaces. Thus for a composite surface with two roughness scales, the first slightly rough and the second very rough and with the longer correlation distance, the scattering cross section accounts for both Bragg scattering and specular scatter. However, the contribution due to specular scatter, by the very rough surface, is slightly damped as a result of the superimposed slightly rough surface. Shadowing effects have been accounted for explicitly in the analysis. If the correlation distance  $\ell_i$  is not the distinguishing feature of the different rough surface heights  $h_i$ , the starting point of the analysis for the composite surface is (38). Since (43) assumes that the correlation distance is the distinguishing feature of the different rough surface heights  $h_i$  (37), it cannot be used in general, and the cross section for the composite surface is not given by a superposition of the cross sections for the surface heights  $h_i$  (44), (45) or (56).

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Figure Caption

Fig. 1. Plane of incidence, scattering plane and reference x,z plane.

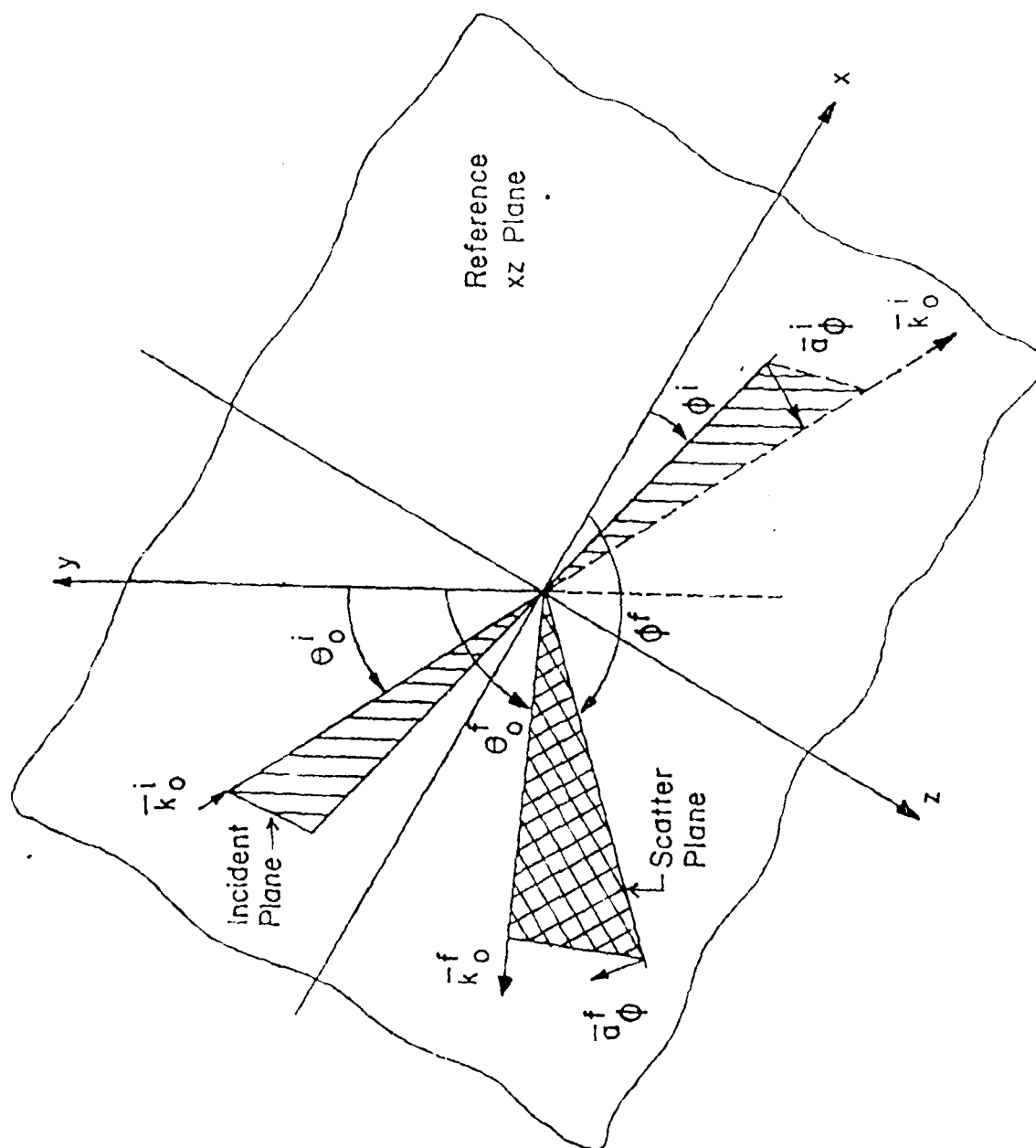


FIG. 1.

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